Numerical implementation of the mixed potential integral equation for planar structures with ferrite layers arbitrarily magnetized

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[1] This work presents a new implementation of the mixed potential integral equation (MPIE) for planar structures that can include ferrite layers arbitrarily magnetized. The implementation of the MPIE here reported is carried out in the space domain. Thus it will combine the well-known numerical advantages of working with potentials as well as the flexibility for analyzing nonrectangular shape conductors with the additional ability of including anisotropic layers of arbitrarily magnetized ferrites. In this way, our approach widens the scope of the space domain MPIE and sets this method as a very efficient and versatile numerical tool to deal with a wide class of planar microwave circuits and antennas.

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1. Introduction

[2] The use of microwave ferrite materials is well known to provide the nonreciprocal characteristics required in some microwave devices as well as tuning capabilities through the application of an external magnetic field [*Baden Fuller*, 1987; *Schuster and Luebbers*, 1996; *Xie and Davis*, 2001]. The inclusion of ferrite layers in planar transmission lines, planar circuits and planar antennas has been object of attention by a number of researchers [*Pozar and Sanchez*, 1988; *Pozar*, 1992; *Yang*, 1994; *Fukusako and Tsutsumi*, 1997; *Tsang and Langley*, 1998; *Oates and Dionne*, 1999; *How et al.*, 2000; *Nurgaliev et al.*, 2001; *León et al.*, 2001, 2002]. Unfortunately, most of the common computer tools currently employed for the analysis and design of planar printed circuits and antennas cannot be applied to structures whose layered substrate includes nonisotropic materials. Nevertheless, a spectral domain implementation of the electric field integral equation (EFIE) [see, e.g., *Pozar*, 1992; *León et al.*, 2002] is available to deal with planar structures loaded with ferrite layers. Indeed, the inclusion of nonisotropic layers is relatively straightforward in the spectral domain frame since spectral domain Green's functions have been developed for general linear media, including ferrites. However, a clear disadvantage of the spectral domain approach lies on its inability to handle efficiently with nonrectangular shape conductors. This limitation can be very important in practice and strongly reduces the versatility of the numerical tools based on that approach.

[3] The incorporation of nonrectangular shaped conductors requires to use space domain formulations, which are suitable for using basis functions that can match any geometry. Thus a possible solution of the aforementioned problem could be the implementation of the corresponding EFIE in the space domain after performing the necessary inverse Fourier transformations to obtain the space domain counterpart of the spectral domain Green's dyadic. However, the space domain Green's dyadic required to solve the EFIE (for both isotropic and/or anisotropic structures) presents hypersingularities [*Bressan and Conciauro*, 1985; *Tai*, 1971], which are further transferred to the reaction integrals appearing after application of the method of moments (MOM) to solve the integral equation [*Arcioni et al.*, 1997]. The presence of these hypersingularities in the reaction integrals clearly degrades the numerical performance of the method and makes it necessary a lot of previous analytic preprocessing. This preprocessing has been already carried out in the case of using only

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isotropic and/or some kind of nonisotropic substrates (for example, uniaxial dielectrics). In such situations the above hypersingularities have been conveniently treated by the authors, thus making the space domain EFIE as competitive numerical tool as the alternative mixed potential integral equation (MPIE) in those circumstances [*Plaza et al.*, 2002; *Mesa and Medina*, 2002]. Unfortunately, the techniques reported by *Plaza et al.* [2002] and *Mesa and Medina* [2002] cannot be easily extended to deal with more general types of anisotropy. In particular, it has been the considerable difficulty to find a closed-form expression for the quasi-static part of the spectral domain Green's dyadic in the case of general anisotropy what has precluded the obtaining of explicit and closed-form expressions for the hypersingular terms of the corresponding EFIE space domain Green's dyadic [*Plaza et al.*, 2002].

[4] Nevertheless, there is still another possibility. Indeed, a convenient solution to the problem under discussion would be the implementation in the space domain of a MPIE (which is free of hypersingularities) that could also deal with complex nonisotropic layers. This purpose seems to be feasible, at least for planar structures whose layered substrate presents any type of magnetic anisotropy, once a numerical method to compute the required space domain Green's functions associated with the MPIE has been reported [*Mesa and Medina*, 2004]. Thus, starting from the Green's functions reported by *Mesa and Medina* [2004], the present paper will extend the work of *Mesa and Medina* [2005] presenting the details of the explicit implementation and numerical solution of the MPIE for planar structures having metallizations of arbitrary shape and layers of isotropic/uniaxially anisotropic dielectrics and/or ferrites magnetized by an external biasing field arbitrarily oriented. The power of the method is illustrated by means of the simulation of planar filters printed on magnetized ferrite substrates.

2. Analysis

[5] The problem of a printed planar structure with a layered substrate that can include isotropic/uniaxial-anisotropic dielectrics and arbitrarily magnetized ferrites is posed in terms of the following MPIE for the tangential electric field, $\mathbf{E}_t$, on the surface of the conductors:

$$\mathbf{E}_t\big|_{\text{cond}} = -\mathrm{j}\omega\mathbf{A}_t[\mathbf{J}] - \nabla_t\Phi\left[\frac{\nabla\cdot\mathbf{J}}{\mathrm{j}\omega}\right] = 0. \qquad (1)$$

where $\mathbf{J}$ is the surface current density on the conductors which are assumed perfect. An harmonic time dependence of the type $\exp(\mathrm{j}\omega t)$ is assumed throughout the paper.

[6] The method of moments (MOM) is now used to solve the above integral equation after expanding the surface current density, $\mathbf{J}$, as

$$\mathbf{J} = \sum_{n=1}^{N} a_n\mathbf{J}_n, \qquad (2)$$

where $\mathbf{J}_n$ are basis functions defined in subsectional triangular regions in order to be able of modeling any conductor shape. The application of the MOM leads to the following system equation:

$$\langle\mathbf{J}_m,\mathbf{E}_t\rangle = \sum_{n=1}^{N} a_n(\Omega_{mn} + \Upsilon_{mn}), \quad m = 1,\ldots,N \qquad (3)$$

where

$$\Omega_{mn} = -\mathrm{j}\omega\langle\mathbf{J}_m, \overline{\mathbf{G}}_A\otimes\mathbf{J}_n\rangle \qquad (4)$$

$$\Upsilon_{mn} = \frac{1}{\mathrm{j}\omega}\langle\mathbf{J}_m, \nabla\Phi[q_n]\rangle, \qquad (5)$$

with $q_n = \nabla\cdot\mathbf{J}_n$, $\langle\cdot,\cdot\rangle$ accounts for inner product, $\overline{\mathbf{G}}_A$ denotes the space domain Green's dyadic that relates the magnetic vector potential with the current density, and $\otimes$ means convolution product.

[7] The application of the divergence theorem to the reaction integrals Υ_{mn} allows us to express equation (5) as

$$\Upsilon_{mn} = \frac{1}{\mathrm{j}\omega}\left\{\int_C \Phi_n\mathbf{J}_m\hat{\mathbf{n}}\,\mathrm{d}l - \int_S q_m\Phi[q_n]\mathrm{d}S\right\}, \qquad (6)$$

where $\hat{\mathbf{n}}$ is the unit vector normal to the path C that surrounds the surface region S where the basis function $\mathbf{J}_m$ is defined. The contribution of the linear integral term in equation (6) can be ignored since either it gets null at the exterior edges of the conductor boundaries or it is eventually canceled out by an opposite term in the interior edges. Thus the finally relevant contribution of equation (6) can be expressed as

$$\Upsilon_{mn} = -\frac{1}{\mathrm{j}\omega}\langle q_m, G_\Phi\otimes q_n\rangle, \qquad (7)$$

where G_Φ is the space domain Green's function that relates the scalar potential with the surface charge.

[8] If the well-known triangular subdomain RWG functions [*Rao et al.*, 1982] are employed as basis functions, it is found that $q_m \equiv q_n = 2$, and the reaction

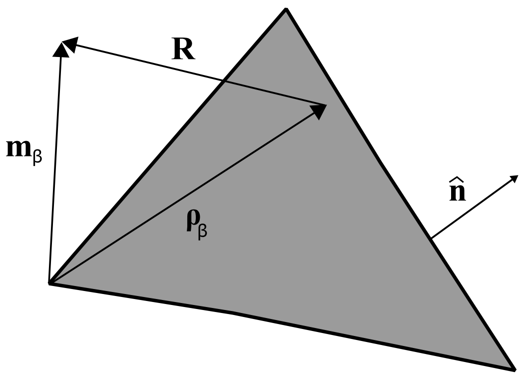

Figure 1. Geometry related to integral $I_{\alpha\beta}^{pq}$.

integrals (4) and (7) can be readily obtained from the following integrals:

$$\Omega_{\alpha\beta}^{pq} = -\frac{\mathrm{j}\omega}{h_\alpha h_\beta}\int_{T_p} \mathrm{d}S\ \boldsymbol{\rho}_\alpha(\mathbf{r})\cdot\int_{T_q'} \mathrm{d}S'\ \overline{\mathbf{G}}_A(\mathbf{r}-\mathbf{r}')\cdot\boldsymbol{\rho}_\beta(\mathbf{r}') \tag{8}$$

$$\Upsilon_{\alpha\beta}^{pq} = -\frac{4}{\mathrm{j}\omega h_\alpha h_\beta}\int_{T_p}\mathrm{d}S\int_{T_q'}\mathrm{d}S'\ G_\Phi(|\mathbf{r}-\mathbf{r}'|), \tag{9}$$

where T_s denotes the triangular subdomain s, and the other geometrical quantities can be referred, for example, to *Mesa and Medina* [2002, Figure 2].

[9] Before to deal with the computation of the above reaction integrals, the vector potential and the scalar potential Green's functions have to be obtained. This topic has been widely treated in the literature [*Michalski and Zheng*, 1990a, 1990b; *Sercu et al.*, 1995] but not for the case of arbitrarily magnetized ferrite layers. Only recently [*Mesa and Medina*, 2004] a method able to compute the MPIE Green's functions for the case of planar structures with magnetic anisotropic layers has been reported. In that work, the space domain MPIE Green's functions were computed by performing an inverse double Fourier transform of the corresponding spectral domain counterparts, which in turn were derived from the EFIE dyadic Green's function. Following [*Mesa and Medina*, 2004], the regular parts of the MPIE Green's functions have to be computed by means of an intensive double numerical Fourier integration whereas the singular parts of these functions can be expressed in closed form as

$$\overline{\mathbf{G}}_{A,\mathrm{sing}}(\rho,\varphi) = \frac{\overline{\boldsymbol{\Gamma}}(\varphi+\pi/2)}{2\pi\rho} \tag{10}$$

$$\overline{\mathbf{G}}_{\Phi,\mathrm{sing}}(\rho) = \frac{\Psi}{2\pi\rho}\ , \tag{11}$$

where ρ, φ are the polar coordinates in the tangential plane, and $\overline{\Gamma}$ and Ψ are related to the asymptotic values of the spectral domain Green's functions in the following way:

$$\overline{\boldsymbol{\Gamma}}(\xi) = \lim_{k_\rho\to\infty} k_\rho \widetilde{\overline{\mathbf{G}}}_A\left(k_\rho,\xi\right) \tag{12}$$

$$\Psi = \lim_{k_\rho\to\infty} k_\rho \widetilde{G}_\Phi\left(k_\rho\right) \tag{13}$$

(k_ρ and ξ are the radial and the angular spectral variables respectively).

[10] The decomposition of the Green's functions in regular and singular parts is further translated to the computation of the reaction integrals, which allows us to write

$$\Omega_{\alpha\beta}^{pq} = \Omega_{\alpha\beta,\mathrm{reg}}^{pq} + \Omega_{\alpha\beta,\mathrm{sing}}^{pq} \tag{14}$$

$$\Upsilon_{\alpha\beta}^{pq} = \Upsilon_{\alpha\beta,\mathrm{reg}}^{pq} + \Upsilon_{\alpha\beta,\mathrm{sing}}^{pq}\ . \tag{15}$$

The regular parts above are numerically computed by means, for example, of appropriate Stroud triangular quadratures [*Stroud*, 1971; *Graglia*, 1993], adjusting the number of quadrature points in function of the distance between the triangular subdomains T_p and T_q. In our computer codes, a single quadrature point is used if the distance between subdomains is larger than λ_0 (free-space wavelength), three points are used if that distance ranges between $0.1\lambda_0$ and λ_o and seven points if the triangular subdomains are closer than $0.1\lambda_0$. The double surface integrals related to the singular part of the scalar Green's function,

$$\Upsilon_{\alpha\beta,\mathrm{sing}}^{pq} = -\frac{4}{\mathrm{j}\omega h_\alpha h_\beta}\int_{T_p}\mathrm{d}S\int_{T_q'}\mathrm{d}S'\frac{1}{R}\ , \tag{16}$$

(R is the modulus of vector $\mathbf{R}$ in Figure 1) can be computed following, for example, the procedures presented by *Wilton et al.* [1984], *Graglia* [1993], *Arcioni et al.* [1997], and *Rossi and Cullen* [1999]. The singular integrals related to the vector potential Green's function can be expressed as

$$\Omega_{\alpha\beta,\mathrm{sing}}^{pq} = -\frac{\mathrm{j}\omega}{h_\alpha h_\beta} I_{\alpha\beta}^{pq}\ , \tag{17}$$

where

$$I_{\alpha\beta}^{pq} = \int_{T_p}\mathrm{d}S\ \boldsymbol{\rho}_\alpha(\mathbf{r})\cdot\int_{T_q'}\mathrm{d}S'\frac{\overline{\mathbf{Q}}(\varphi)}{R}\cdot\boldsymbol{\rho}_\beta(\mathbf{r}'), \tag{18}$$

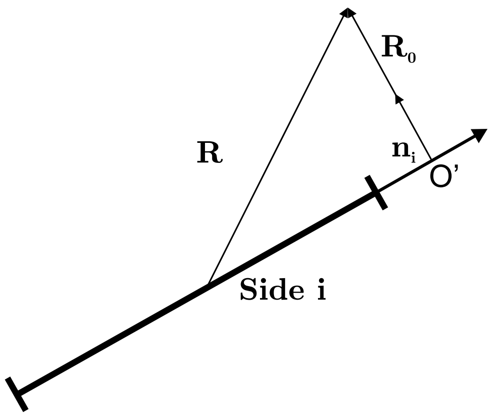

Figure 2. Geometry related to the contour integrals in each side of the triangle.

with $\overline{\mathbf{Q}}\,(\varphi) \equiv \overline{\mathbf{\Gamma}}(\varphi + \pi/2)$. At this point it is interesting to note that for structures with cylindrical symmetry with respect to the normal-to-interfaces axis (i.e., structures with layers of isotropic and uniaxially anisotropic dielectrics, as well as ferrites with the external magnetization along the normal axis), the singular integrals to be treated are simplified forms of equation (18). In particular, they are found to be of the following general type:

$$I^{pq}_{\alpha\beta} = \int_{T_p} \mathrm{d}S\; \boldsymbol{\rho}_\alpha \cdot \int_{T'_q} \mathrm{d}S' \frac{1}{R}\; \boldsymbol{\rho}_\beta \;, \qquad (19)$$

which has been conveniently treated in the literature [*Arcioni et al.*, 1997; *Mesa and Medina*, 2002]. In our case, the presence of the dyadic appearing in the integrand of equation (18) as well as it angular dependence precludes the use of some of the efficient schemes (and closed-form expressions) previously reported. In this way, and after trying different approaches based on analytical preprocessing, our most efficient procedure to compute equation (18) involves to write the vector from vertex β of triangle T'_q (see Figure 1) as

$$\boldsymbol{\rho}_\beta = \mathbf{m}_\beta - \mathbf{R} \;, \qquad (20)$$

with $\mathbf{m}_\beta$ being constant when integrating in T'_q. This trick allows us to express equation (18) as

$$I^{pq}_{\alpha\beta} = \int_{T_p} \mathrm{d}S\; \boldsymbol{\rho}_\alpha \cdot \left[\overline{\mathbf{I}}_4 \cdot \mathbf{m}_\beta - \mathbf{F}\right] \;, \qquad (21)$$

where

$$\overline{\mathbf{I}}_4 = \int_{T'_q} \mathrm{d}S' \frac{\overline{\mathbf{Q}}(\varphi)}{R} \qquad (22)$$

$$\mathbf{F} = \int_{T'_q} \mathrm{d}S'\; \overline{\mathbf{Q}}(\varphi) \cdot \hat{\mathbf{R}} \;, \qquad (23)$$

where $\hat{\mathbf{R}} = \mathbf{R}/R$. The integrand of the surface integral $\mathbf{F}$ above always shows a smooth behavior, and thus this integral can be numerically performed very efficiently using, for example, Stroud quadratures of low order (in practice seven-point quadratures are found to provide sufficient accuracy). In order to compute the dyadic singular surface integral in equation (22), the following identity has been used:

$$\frac{Q_{ij}(\varphi)}{R} = Q_{ij}(\varphi) \nabla \cdot \hat{\mathbf{R}} = \nabla \cdot \left[Q_{ij}(\varphi)\hat{\mathbf{R}}\right] \;, \qquad (24)$$

so as to turn equation (22) into a contour integral after applying the divergence theorem:

$$I_{4,ij} = -\int_{T'_q} \mathrm{d}S'\; \nabla' \cdot \left[Q_{ij}(\varphi)\hat{\mathbf{R}}\right] \qquad (25)$$

$$= -\int_{\partial T'_q} \mathrm{d}l'\; Q_{ij}(\varphi)\hat{\mathbf{n}} \cdot \hat{\mathbf{R}} \;. \qquad (26)$$

[11] Operating in the contour integral (26), and taking into account the geometry shown in Figure 2, $\overline{\mathbf{I}}_4$ can be finally expressed as

$$\overline{\mathbf{I}}_4 = -\sum_{i=1}^{3} \mathbf{R}_{0,i} \cdot \hat{\mathbf{n}}_i \int_{\mathrm{side}\; i} \frac{\overline{\mathbf{Q}}(\varphi)}{R}\; \mathrm{d}l' \;. \qquad (27)$$

Fortunately, this final form of the integral can be efficiently performed by means of, for example, Gauss-

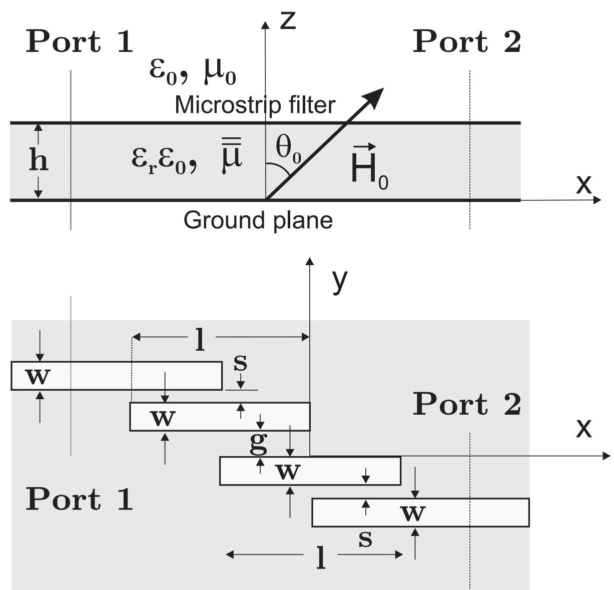

Figure 3. Layout of the coupled line filter printed on a ferrite substrate analyzed by *León et al.* [2004] and used for comparison purposes.

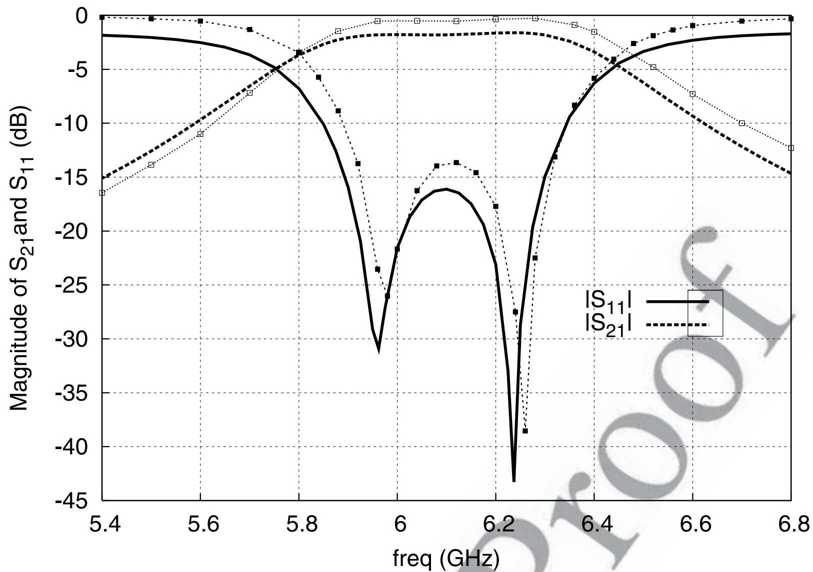

Figure 4. Return ($|S_{11}|$) and insertion ($|S_{21}|$) losses of coupled line microstrip filter printed on a normally magnetized ferrite with (see Figure 3) w = 0.38 mm, s = 0.19 mm, g = 0.76 mm, l = 7.8 mm, h = 0.625 mm, ε_r = 15, $\mu_0 M_s$ = 0.178 T, $\mu_0 H_0$ = 0.01 T, and $\mu_0 \Delta H$ = 0.001 T. Open and solid squares correspond to the results reported by *León et al.* [2004].

Kronrod quadratures of low order. Note that point O' might be a point of the side i. In case O' belongs to side i, it has been found very convenient to split the integration interval into two intervals so as to avoid the associated quasi-singularity appearing in the integrand.

3. Numerical Results

[12] In this section some results will be shown to validate the accuracy of the present approach and to illustrate its potential as a useful tool to analyze relatively complex structures containing layers of magnetized ferrites.

[13] The validation of the present approach will be done by comparing our results with those previously obtained by *León et al.* [2004] for a coupled line microstrip filter fabricated on a layer of magnetized ferrite. The layout of the filter is shown in Figure 3. The analysis of *León et al.* [2004] is carried out in the spectral domain, for which the structures there studied only involved rectangular-shaped conductors. In particular, the comparison in Figure 4 shows that our results for the return and

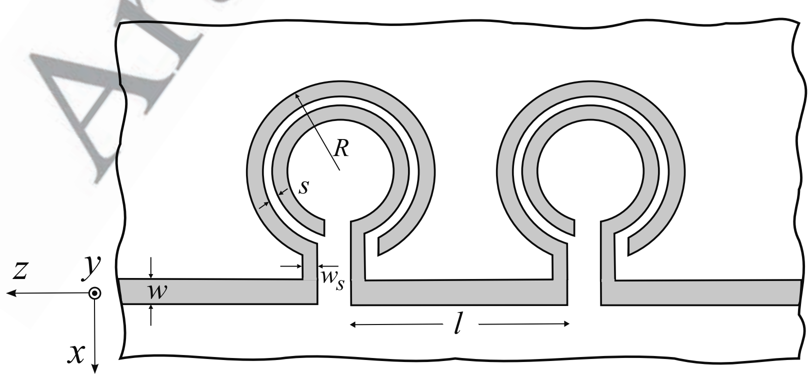

Figure 5. Top view of a pair of SRRs printed on a grounded ferrite slab and excited by a microstrip line. For structural parameter of the substrate, h = 0.49 mm, ε_r = 15, $\mu_0 M_s$ = 0.8 T, $\mu_0 H_0$ = 0.2 T, and $\mu_0 \Delta H$ = 0.001 T; for the microstrip line, w = 0.3 mm; and for the SRR, R = 2.2 mm, w_s = 0.14 mm, s = 0.25 mm, and l = 4.25 mm.

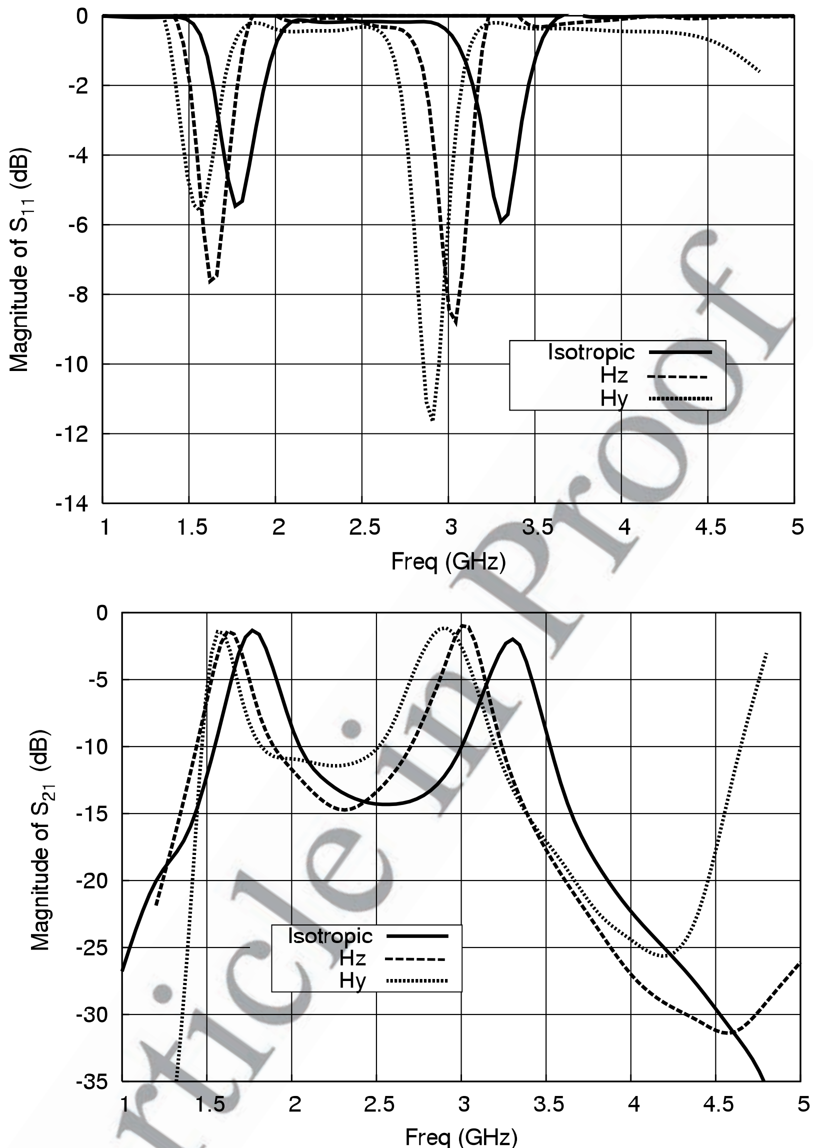

Figure 6. (top) Return and (bottom) insertion losses (in dB) of a pair of SRRs excited by a microstrip line. The structural parameters are given in Figure 5.

insertion losses of the filter printed on a normally biased ferrite agree reasonably well with those reported by *León et al.* [2004, Figure 3]. The same agreement has been found (although not explicitly shown) for other values of the external magnetizing field. Differences can be attributed to the different meshing schemes used in the compared methods. This favorable comparison means that the computer code based on the new space domain formulation presented in this paper can be used with confidence.

[14] Once our approach has been validated for the case of rectangular shaped structures, it is now used to study a geometrically more complex nonrectangular planar circuit. In particular we will study the possibilities of external tunability of the characteristics of a compact dual-band microstrip filter built up with split ring resonator (SRR) particles. Our study will focus on the analysis of a pair of SRRs excited by a microstrip line as shown in Figure 5.

[15] This basic structure is a derivation on a filter previously reported by *Martel et al.* [2004], with the difference that our structure does not have a window in the ground plane. The structure under analysis behaves as a nonoptimized dual-band filter, and it could be the

basis for more practical designs containing SRR particles after applying an optimization process (this topic is beyond the scope of the present work, although the method here presented should be a convenient tool for this purpose). Thus Figure 6 shows the effect of an external magnetic field biasing the structure in two orthogonal directions (along the microstrip direction, z axis, and normally to the interfaces, y axis) on the return and insertion losses of the structure. These results are shown together with those corresponding to the same SRR configuration but with the ferrite substrate replaced by a simple dielectric with $\varepsilon_r = 15$. It can be observed how the external magnetization field clearly provides a method to tune the passbands of the filter. Tuning can be carried out by adjusting the intensity of the magnetic field or its direction with respect to the normal to the ferrite substrate.

4. Conclusions

[16] This paper has presented a new implementation of the MPIE in the space domain able to deal with planar structures containing anisotropic magnetic layers and conductors of arbitrary shape. The corresponding space domain Green's functions (previously developed by the authors) are the kernel of the integral equation, whose solution by means of the MOM gives rise to a new type of reaction integrals that are here treated and their computation optimized. Some results are shown to validate our proposal, and finally some new results are presented for a pair of split ring resonators printed on a grounded ferrite excited by a microstrip line. This structure can be an example of the potentiality of the method for the design of tunable filters and other devices by means of an external biasing magnetic field.

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References

Arcioni, P., M. Bressan, and L. Perregrini (1997), On the evaluation of the double surface integrals arising in the application of the boundary integral method to 3-D problems, *IEEE Trans. Microwave Theory Tech.*, *45*, 436–439.

Baden Fuller, A. J. (1987), *Ferrites at Microwave Frequencies*, Peter Peregrinus, London.

Bressan, M., and G. Conciauro (1985), Singularity extraction from the electric Green's function for a spherical resonator, *IEEE Trans. Microwave Theory Tech.*, *33*, 407–414.

Fukusako, T., and M. Tsutsumi (1997), Superconducting microstrip resonator with yttrium iron garnet single crystal, *IEEE Trans. Microwave Theory Tech.*, *45*, 2013–2017.

Graglia, R. D. (1993), On the numerical integration of the linear shape function times the 3-D Green's functions or its gradient on a plane triangle, *IEEE Trans. Antennas Propag.*, *41*, 1448–1455.

How, H., P. Shi, C. Vittoria, E. Hokanson, M. N. Champion, L. C. Kempel, and K. D. Trott (2000), Steerable phased array antennas using single-crystal YIG phase shifters: Theory and experiments, *IEEE Trans. Microwave Theory Tech.*, *48*, 1544–1549.

León, G., R. R. Boix, and F. Medina (2001), Efficient full-wave characterization of microstrip lines fabricated on magnetized ferrites with arbitrarily oriented bias field, *J. Electromagn. Waves Appl.*, *15*, 223–251.

León, G., R. R. Boix, and F. Medina (2002), Full-wave analysis of a wide class of microstrip resonators fabricated on magnetized ferrites with arbitrarily oriented bias magnetic field, *IEEE Trans. Microwave Theory Tech.*, *50*, 1510–1519.

León, G., R. R. Boix, and F. Medina (2004), Tunability and bandwidth of microstrip filters fabricated on magnetized ferrites, *IEEE Microwave Wireless Components Lett.*, *14*, 171–173.

Martel, J., R. Marqués, F. Falcone, J. D. Baena, F. Medina, F. Martín, and M. Sorolla (2004), A new LC series element for compact bandpass filter design, *IEEE Microwave Wireless Components Lett.*, *14*, 210–212.

Mesa, F., and F. Medina (2002), Efficient evaluation of reaction integrals in the EFIE analysis of planar layered structures with uniaxial anisotropy, *IEEE Trans. Microwave Theory Tech.*, *50*, 2142–2146.

Mesa, F., and F. Medina (2004), Numerical computation of the space-domain mixed potential Green's functions for planar layered structures with arbitrarily magnetized ferrites, *IEEE Trans. Antennas Propag.*, *52*, 3019–3025.

Mesa, F., and F. Medina (2005), Solution of the EFIE for printed circuits on ferrite substrates, paper presented at 2005 IEEE Antennas and Propagation Society International Symposium, Inst. of Electr. and Electron. Eng., Washington, D. C.

Michalski, K. A., and D. Zheng (1990a), Electromagnetic scattering and radiation by surfaces of arbitrary shape in layered media, part I: Theory, *IEEE Trans. Antennas Propag.*, *38*, 335–352.

Michalski, K. A., and D. L. Zheng (1990b), Electromagnetic scattering and radiation by surfaces of arbitrary shape in layered media, part II: Implementation and results for contiguous half-spaces, *IEEE Trans. Antennas Propag.*, *38*, 345–352.

Nurgaliev, T., S. Miteva, A. Jenkins, and D. Dew-Hughes (2001), Transmission characteristics of HTS microstrip resonators with a ferrite component, *IEEE Trans. Appl. Superconductivity*, *11*, 446–449.

Oates, D. E., and G. F. Dionne (1999), Magnetically tunable superconducting resonators and filters, *IEEE Trans. Appl. Superconductivity*, *9*, 4170–4175.

Plaza, G., F. Mesa, and F. Medina (2002), Treatment of singularities and quasi-static terms in the EFIE analysis of planar structures, *IEEE Trans. Antennas Propag.*, *50*, 191–485.

Pozar, D. M. (1992), Radiation and scattering characteristics of microstrip antennas on normally biased ferrite substrates, *IEEE Trans. Antennas Propag.*, *40*, 1084–1092.

Pozar, D. M., and V. Sanchez (1988), Magnetic tuning of a microstrip antenna on a ferrite substrate, *Electron. Lett.*, *24*, 729–731.

Rao, S. M., D. R. Wilton, and A. W. Glisson (1982), Electromagnetic scattering by surfaces of arbitrary shape, *IEEE Trans. Antennas Propag.*, *30*, 409–418.

Rossi, L., and P. J. Cullen (1999), On the fully numerical evaluation of the linear-shape function times the 3-D Green's function on a plane triangle, *IEEE Trans. Microwave Theory Tech.*, *47*, 398–402.

Schuster, J. W., and R. J. Luebbers (1996), Finite difference time domain analysis of arbitrarily biased magnetized ferrites, *Radio Sci.*, *31*, 923–929.

Sercu, J., N. Faché, F. Libbrecht, and P. Lagase (1995), Mixed potential integral equation technique for hybrid microstrip-slotine multilayered circuits using a mixed rectangular-triangular mesh, *IEEE Trans. Microwave Theory Tech.*, *43*, 1162–1172.

Stroud, A. (1971), *Approximate Calculation of Multiple Integrals*, Prentice-Hall, Upper Saddle River, N. J.

Tai, C.-T. (1971), *Dyadic Green's Functions in Electromagnetic Theory*, Intext Educ. Publ., Scranton, Pa.

Tsang, K. K., and R. J. Langley (1998), Design of circular patch antennas on ferrite substrates, *Proc. Inst. Electr. Eng., Part H*, *145*, 49–55.

Wilton, D. R., S. M. Rao, A. Glisson, D. Schaubert, O. Al-Bundak, and C. Butler (1984), Potential integrals for uniform and linear source distributions on polygonal and polyhedral domains, *IEEE Trans. Antennas Propag.*, *32*, 276–281.

Xie, K., and L. E. Davis (2001), Performance of axially magnetized ferrite coupled lines, *Radio Sci.*, *36*, 1353–1361.

Yang, H. Y. (1994), Microstrip open-end discontinuity on a nonreciprocal ferrite substrate, *IEEE Trans. Microwave Theory Tech.*, *42*, 2423–2428.

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