Discrete breathers collisions: An overview and some recent results

Jesús Cuevas Maraver

Nonlinear Physics Group
Universidad de Sevilla

Outline

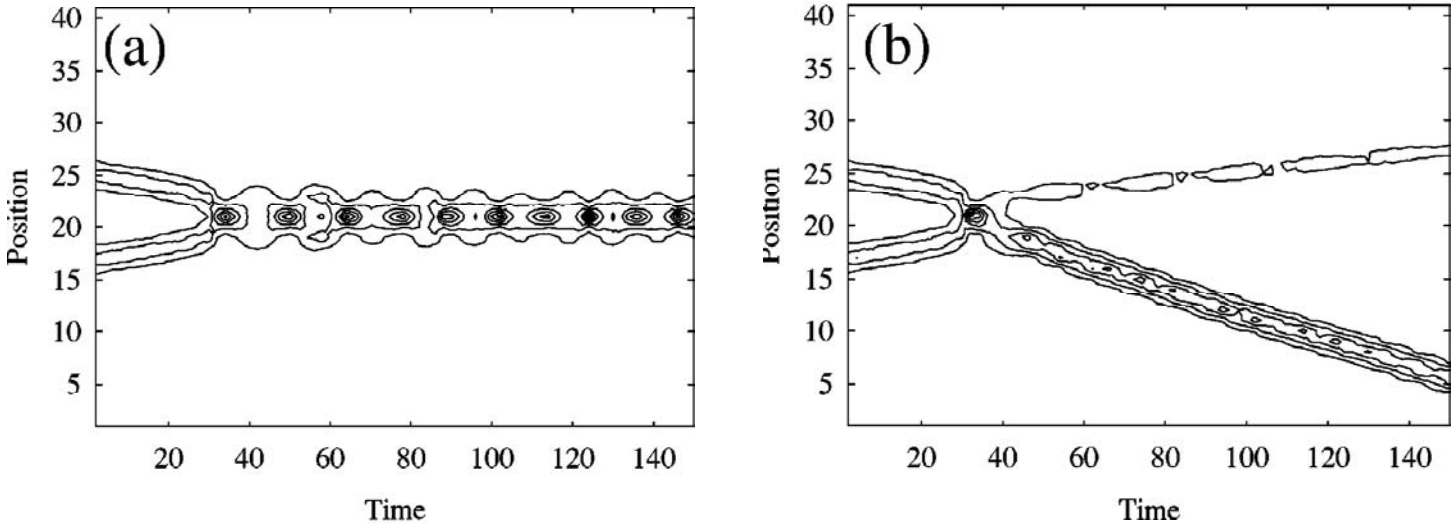
- Overview of previous studies:
  - Collisions in FPU lattices
  - Collisions in dissipative Klein–Gordon lattices
  - Collisions in DNLS lattices
- New results on DNLS lattices with saturable nonlinearity:
  - Properties of SDNLS lattices
  - Collisions in SDNLS lattices
- Future challenges

FPU lattices

- Y. Doi, PRE 68, 066608 (2003)
- β-FPU model is considered
- Collisions of breathers with the same/different energies and the same/different phase
- Observed regimes:
  - Reflection
  - Fusion
  - Symmetry breaking (even if same energy and same phase)
- The energy exchange is throughoutly studied
- Transferred energy may be very sensitive to phase difference

FPU lattices

- Fusion and symmetry breaking regimes

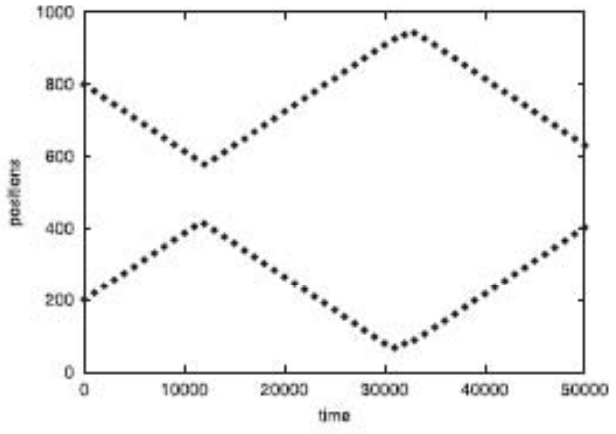


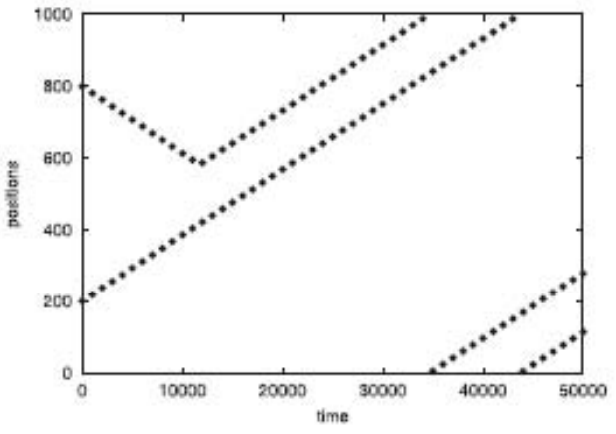
Dissipative Klein-Gordon lattices

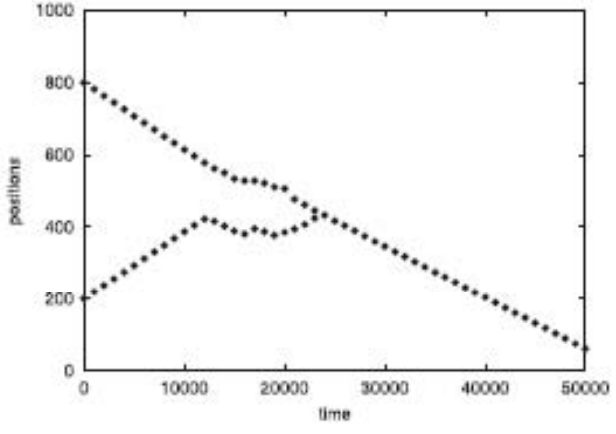
- M. Meister & L.M. Floría, EPJB 37, 213 (2004)
- Frenkel–Kontorova model with dissipation and external force
- Collisions of breathers with the same energy and phase
- Observed regimes:
  - Reflection
  - Destruction of one/both breathers
  - Bound state:
    - Pair of breathers subject to the force of the emitted phonons
    - The distance between breathers is related to the velocity
- The final states are attractors of the system

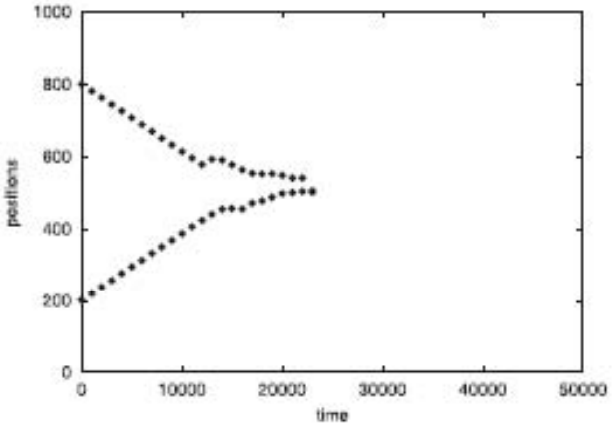
Dissipative Klein-Gordon lattices

- Different regimes



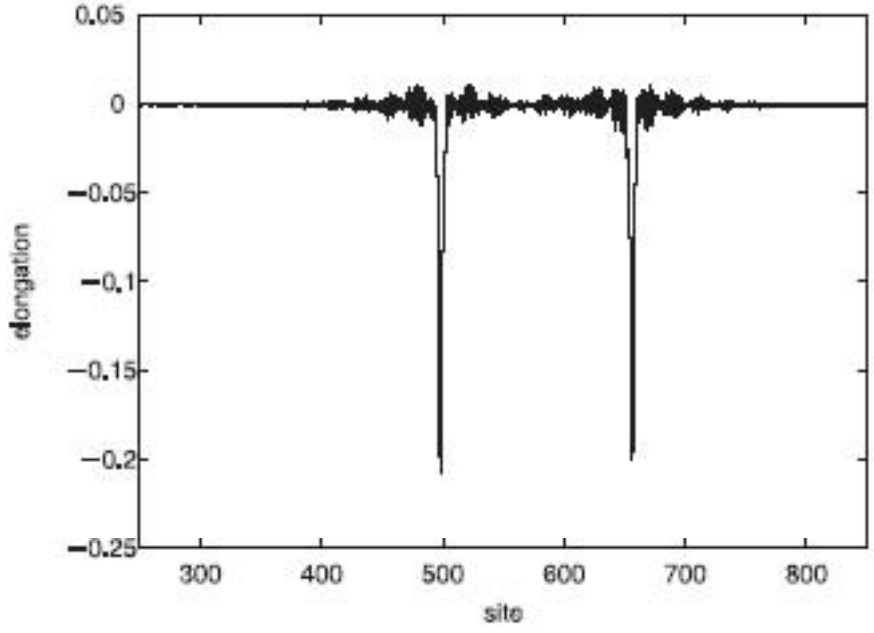






Dissipative Klein-Gordon lattices

- A bound state

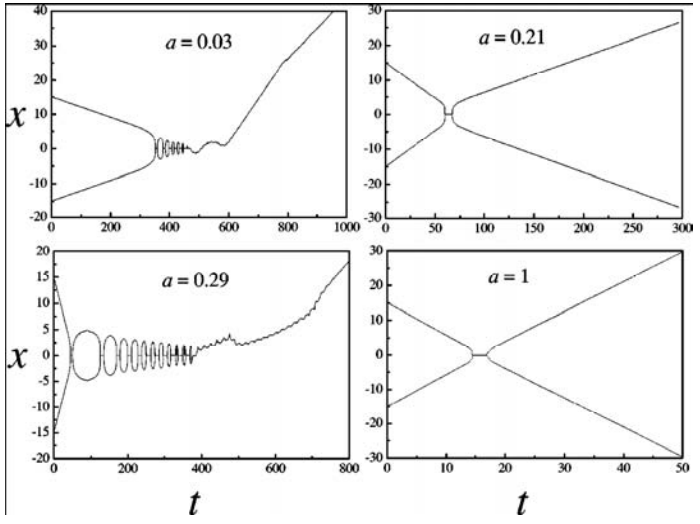


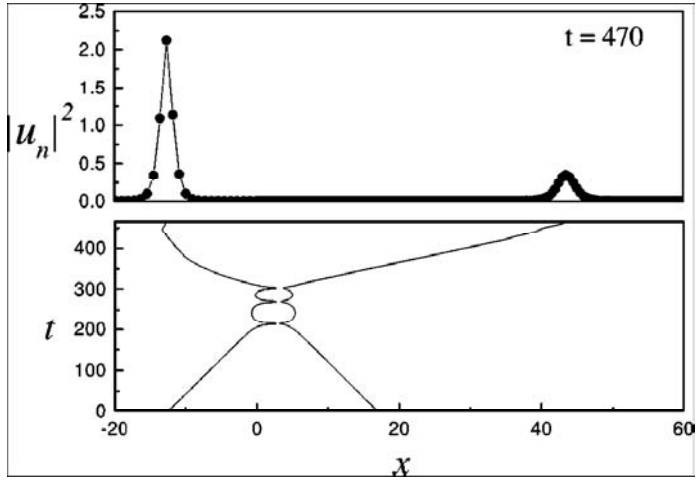
Nonintegrable DNLS lattices

- I.E. Papacharalampous *et al*, PRE 68, 046604 (2003)
- Collisions of breathers with the same energy and the same/different phase
- Observed regimes (in-phase breathers):
  - Reflection
  - Bound state
  - Spontaneous symmetry breaking (due to numerical errors):
    - Bound state moves with a well-defined value of the velocity
    - Mutual bounce after multiple collisions
- High Peierls-Nabarro barrier $\rightarrow$ Strong differences between on-site and inter-site collisions.

Nonintegrable DNLS lattices

- Several regimes





DNLS lattices with saturable nonlinearity

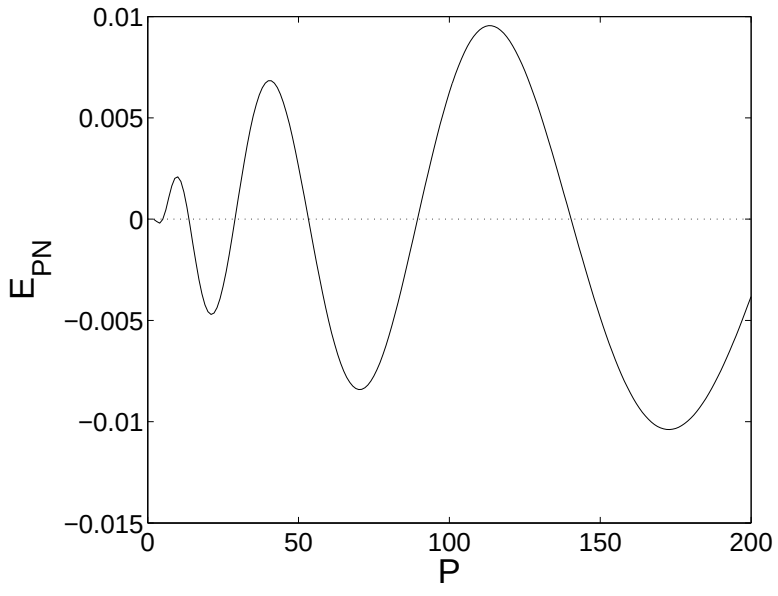
- M. Stepic *et al*, PRE 69, 066618 (2004)
- L. Hadzievski *et al*, PRL 93, 033901 (2004)
- Discrete version of the Vinetskii-Kukhtarev equation (SDNLS equation):

$$i\dot{u}_n - \beta \frac{u_n}{1+|u_n|^2} + (u_{n+1} - 2u_n + u_{n-1}) = 0.$$

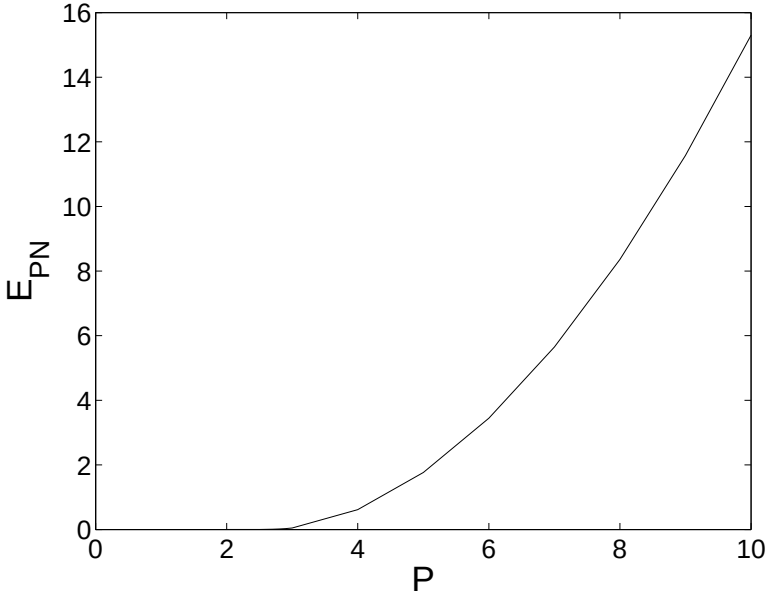
- Models 1D waveguide arrays of photorefractive materials $\rightarrow$ SBN61 ($Sr_{0.61}Ba_{0.39}Nb_2O_6$)
- Main feature: Bounded Peierls-Nabarro barrier $\rightarrow$ High power moving breathers can be found.

DNLS lattices with saturable nonlinearity

- Peierls-Nabarro barrier in 1D chains



Saturable Nonlinearity



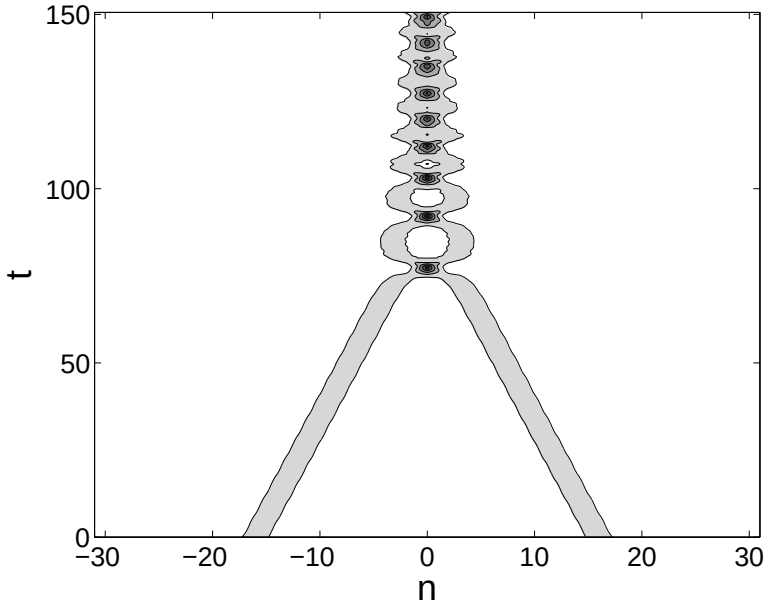
Kerr Nonlinearity

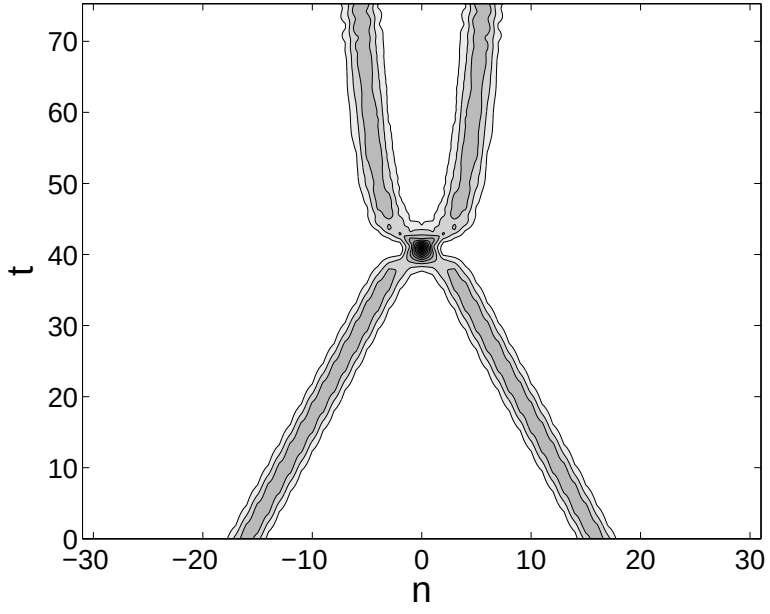
Collisions in SDNLS lattices

- J. Cuevas and J.C. Eilbeck, `ArXiv:nlin.PS/0501050`
- Collisions of breathers with the same energy and phase
- Observed regimes (in-phase breathers):
  - Reflection
  - Bound state
  - Bound state + Reflection (for high powers)
  - Bound state + spontaneous symmetry breaking
- Symmetry breaking due to numerical errors

Collisions in SDNLS lattices

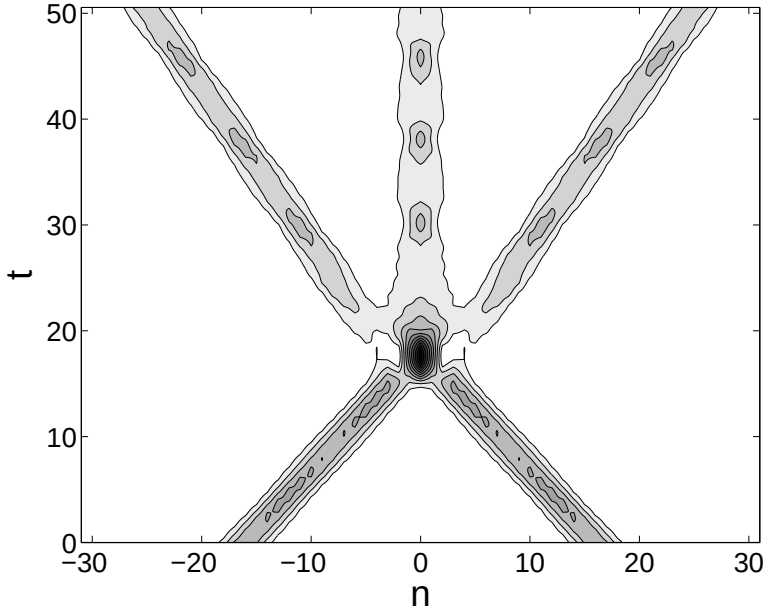
- Reflection and bound state regimes

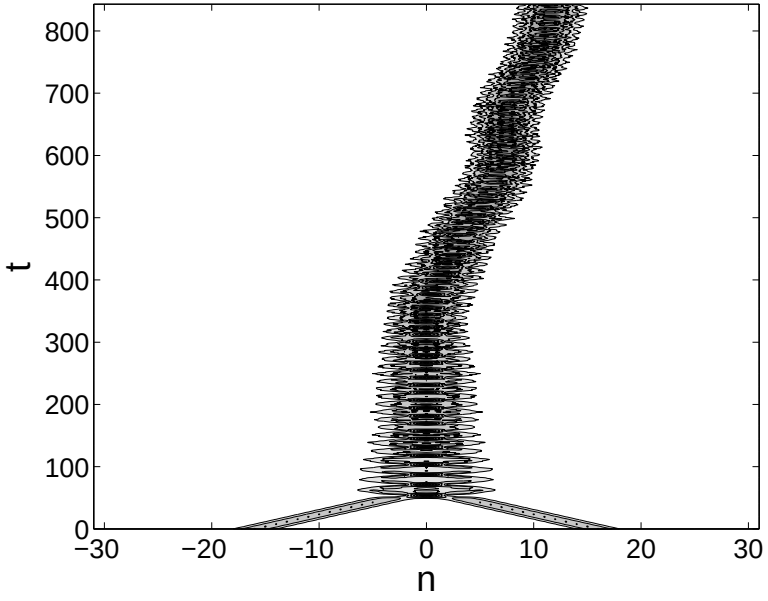




Collisions in SDNLS lattices

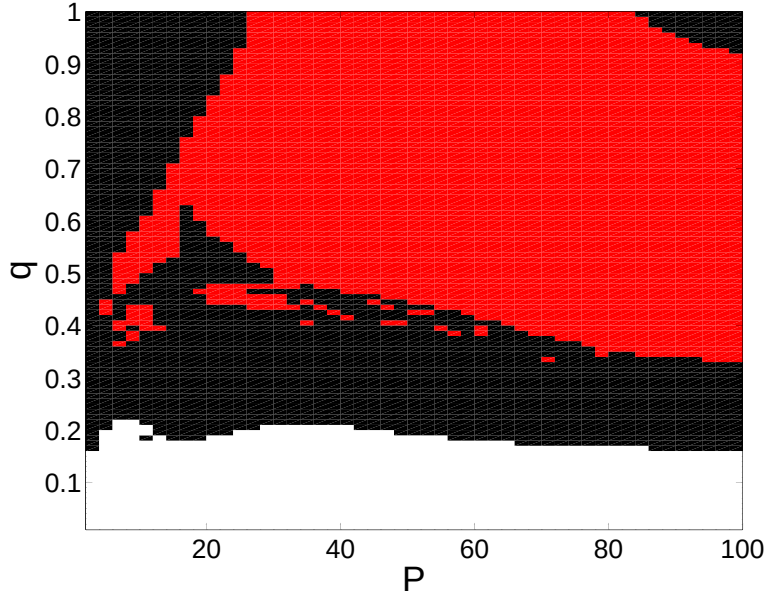
- Reflection + bound state and symmetry breaking regimes



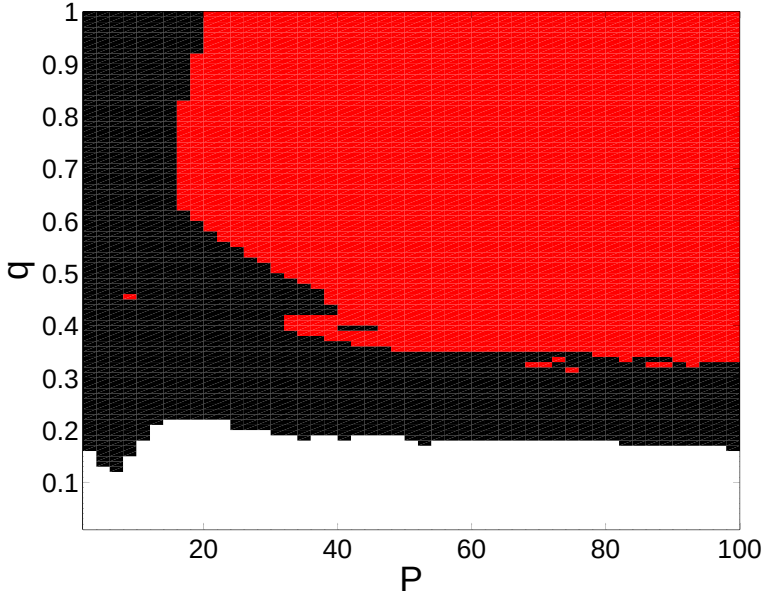


Collisions in SDNLS lattices

- Different regimes



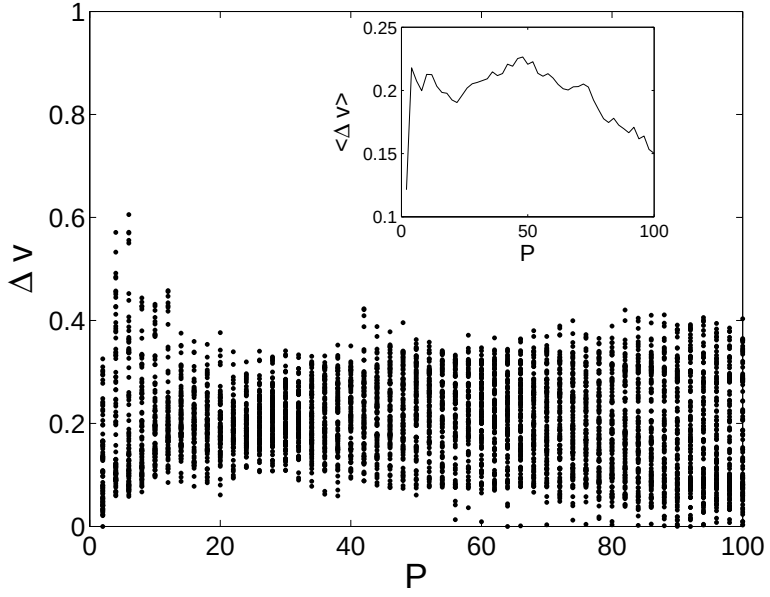
Inter-site collisions



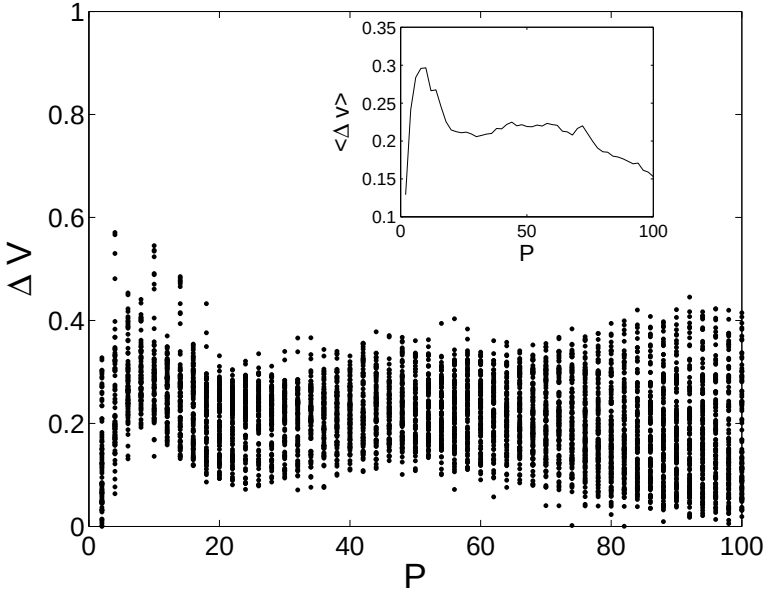
On-site collisions

Collisions in SDNLS lattices

- Velocity difference (Incoming-Outgoing)



Inter-site collisions



On-site collisions

Conclusions

- In all cases, reflection and bound state formation is observed
- Symmetry breaking is also observed, with different origins:
  - DNLS: Numerical errors
  - Dissipative KG: Attractor of the system
  - FPU: Unexplained
- Saturable nonlinearity allows high-power breathers
- Future challenge: Study the case of Hamiltonian Klein-Gordon lattices.