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Kinks in a chain of magnetic coupled pendulums: Experimental and numerical study

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Abstract—We have study numerically and experimentally the propagation of kinks in a chain. Such kind of chain can be, under certain approximations to be consider as a model of a lattice of repelling particles. i.e., a model to experimentally study nonlinear localized travelling waves. We excite pulses at one boundary of the system and demonstrate the existence of transient-kinks, whose dynamics are in good agreement with the theoretical predictions. We also study the dynamics in the presence of an external magnetic substrate potential at the bottom of the chain taht, added to the gravity, mimics the substrate potential in a crystal. The results obtained in the experimental setup can be extrapolated to other systems.

1. Introduction

The basic block-units that form matter are atoms. They interact via electromagnetic forces and form ordered states called lattices. These forces can be repulsive as a result of Coulomb interactions and short range repulsive forces due to the Pauli exclusion principle in many body systems [1]. Then, the study of lattices of repulsive particles can give an insight to certain properties of matter. A motivation for the study of one-dimensional (1D) models in Solid State Physics is the complexity of the full problem. Basic properties of phenomena that occurs at atomic scale can be modelled with the 1D Frenkel-Kontorova (FK) model[2]. Although the literature on FK model is extensive, many problems, however, remain still unsolved[3]. In this work we have performed a numerical and experimental study of kink propagation in a lattice of particles with repulsive interparticle potential and different one-site potentials.

When the lattice is perturbed at its boundaries, waves propagate through them and due to the lattice periodicity strong dispersion appears when wavelengths are of the order of the lattice period together with the nonlinear character of the interparticle interaction. This give rise to wave phenomena like harmonic generation, chain expansion (dilatation) [4] or moving kinks [5, 6, 7, 8] (that behave as particle-like perturbations travelling along the lattice.), that are solutions of the equations that governs the dynamics of the system based on the nonlinear character of the interactions.

2. The model

Our theoretical model consists of an infinite chain of identical particles with mass m aligned along the x-axis. Each particle interacts with its nearest neighbors via a repulsive potential, V_{int}. In the absence of perturbations, every mass has a fixed equilibrium position being the interparticle distance a. An external potential, V_{ext}, can be added to the lattice. In condensed matter, this external potential comes from the action of the other atoms or ions in the crystal. These extra forces can be provided by a periodic on-site potential and/or forces keeping the boundary particles at fixed positions. Then, the equation of motion is written as

$$\ddot{u}_n = -\frac{1}{(1+u_{n+1}-u_n)^{\alpha}} + \frac{1}{(1+u_n-u_{n-1})^{\alpha}} - V'_{\text{ext}} \qquad (1)$$

where u_n represents the displacement of the n-th particle measured with respect to its equilibrium position as shown in Fig. 1 and we consider interparticle forces that decrease with an inverse-power law of the difference between the displacement of neighbours. The term V'_{ext} is the derivative of an external potential. In the limit of small displacements, the interaction force between particles can be approximated linearly with respect to the distance and Eq. 1 represents a system of coupled harmonic oscillators. However, in general the potentials in physical systems are anharmonic. Added to it, as we have mention, we have an external potential V_{ext} and we have study two different configurations:

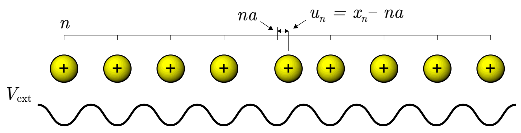

Figure 1: (a) Scheme of the lattice of coupled particles with a substrate potential.

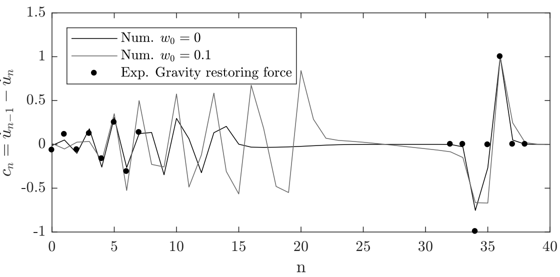

Figure 2: Derivative of the strain corresponding to the case of a transient kink in the chain with the substrate corresponding to the gravitational restoring force ($V'ext = -w0 * un$). The figure shows the comparison between experimental and numerical data.

1. **Harmonic substrate potential**. V'_{ext} is the restoring force of the pendulum,

2. **Periodic substrate potential**: A set of fixed magnets were added at the bottom of the chain, introducing a strong periodic substrate potential, V_{ext}, if compared to the repulsive forces between magnets. The inclusion of this substrate introduces periodic restoring forces with multiple stable states as $u_n = \pm n$: the oscillators can be at rest at different lattice positions.

The experimental results were obtained driving the chain at the first oscillator with a pulsed waveform excitation (half-sinusoidal wave). In particular, we have excited our system using a shaker and detecting the velocity of the pendulums using a laser vibrometer. In Figure 2 we can see an example of the derivative of the strain (experimentally we obtain as a raw data the velocities of pendulums, i.e., de derivative of the displacement) in comparison with the simulated one using Eq.1. We can see that the profile is in agreement with the derivative of a triangular signal corresponding to the profile of the strain of a kink.

In this work we demonstrate the existence of transient kinks in our chain and study the dependence of their velocity as a function of the amplitude of the kink for both, the external potential corresponding to the gravity (i.e. the gravitational force as a restoring force) and the external potential being the gravity plus a repulsive magnetic potential generated by a substrate formed by a periodic distribution of magnets.

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