No Lineal 2018

International Conference on Nonlinear Mathematics and Physics

Málaga, June 12th-15th, 2018

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Book of Abstracts

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TIN2014-56494-C4-1..4-P

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Programa Doctorado UMA de Ingeniería Mecánica y Eficiencia Energética

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The 11th International Conference on Nonlinear Mathematics and Physics (NoLineal 2018) will be held in Málaga (Spain), 12-15th June, 2018. This series of conferences were previously held in Ávila (1997), Almagro (2000), Cuenca (2002), Toledo (2004), Ciudad Real (2007), Barcelona (2008), Cartagena (2010), Zaragoza (2012), Badajoz (2014), Sevilla (2016).
The aim of this conference is to offer senior and young researchers of different areas related to nonlinear sciences, such as Physics, Mathematics, Biology, Economics, Social Sciences, etc, the possibility to share their latest results in this interdisciplinary meeting.

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XI International Conference on Nonlinear Mathematics and Physics
Malaga, June 12th – 15th, 2018
Escuela de Ingenierías Industriales, Sala de Grados A
(Ampliación del Campus de Teatinos)

Program

Tuesday, 12th June

09:00 - 10:15	Registration
10:15 - 10:30	Presentation
10:30 - 11:30	Plenary session: B. Malomed Multidimensional solitons
11:30 - 12:00	Coffee break
12:00 - 13:00	Plenary session: P. Rosenau Compactons in Action: When, Where and How
13:30 - 15:30	Lunch
15:30 - 17:10	Afternoon scientific sessions: lecture room 15:30 A. Durán, 15:55 S. Maza, 16:20 N. Craig, 16:45 I.A. García

Wednesday, 13th June

09:00 - 10:00	Plenary session: R. Carretero Vortices and vortex rings in quantum superfluids: a quasi-particle approach
10:00 - 11:00	Plenary session: J.F.R. Archilla Nonlinear excitations in solids, observation and consequences
11:00 - 11:30	Coffee break
11:30 - 13:35	Morning scientific sessions: lecture room 11:30 J. Valero, 11:55 M. Pulido, 12:20 D. Sinelshchikov, 12:45 R. Caballero, 13:10 M.C. Domínguez
13:40 - 15:30	Lunch
15:30 - 17:10	Afternoon scientific sessions: lecture room 15:30 F. R. Villatoro, 15:55 D. Núñez, 16:20 C.M. García-López, 16:45 M.A. Porras
18:30	Panoramic tour (downtown)

Thursday, 14th June

09:00 - 10:00	Plenary session: Y. Kosevich Phase dynamics of nonlinear localized excitations in two weakly coupled anharmonic chains and analogy with coherent tunneling of quantum objects
10:00 - 11:00	Plenary session: Y. Kartashov Nonlinear phenomena in polariton topological insulators
11:00 - 11:30	Coffee break
11:30 - 13:35	Morning scientific sessions: lecture room 11:30 J. Giné, 11:55 M. Díaz García, 12:20 J.J. Miralles, 12:45 S. V. Dmitriev, 13:10 I. Checa
13:40 - 15:30	Lunch
15:30 - 17:10	Afternoon scientific sessions: lecture room 15:30 J.E. Sandubete, 15:55 T. Dittrich, 16:20 F. Martín-Vergara, 16:45 E. Korznikova
19:00 – 20:00	Popular talk: Manuel Pulido (Rectorate building, downtown) ¿Es reversible el cambio climático antropogénico? Impacto del “agujero de ozono” en el clima del hemisferio sur
20:30	Conference dinner (downtown)

Friday, 15th June

09:00 - 10:00	Plenary session: J. I. Ramos Superdiffusive Burgers-Taylor's kinks and nonlinear dispersive shock waves
10:00 - 11:00	Plenary session: J. C. Losada From 2D to 3D in LiCN: A brief history of Nonlinear Dynamics of a Hamiltonian Molecular System
11:00	Closing coffee

Plenary Talks

Escuela de Ingenierías Industriales, Sala de Grados A
Universidad de Málaga (Ampliación del Campus de Teatinos)

Multidimensional solitons
BORIS MALOMED
Tel Aviv University, Israel

Compactons in Action: When, Where and How
PHILIP ROSENAU
Tel Aviv University, Israel

Vortices and vortex rings in quantum superfluids: a quasi-particle approach
RICARDO CARRETERO
San Diego State University, USA

Nonlinear excitations in solids, observation and consequences
JUAN F. R. ARCHILLA
Universidad de Sevilla, Spain

Phase dynamics of nonlinear localized excitations in two weakly coupled anharmonic chains and analogy with coherent tunneling of quantum objects
YURIY KOSEVICH
Semenov Institute of Chemical Physics, Russian Academy of Sciences, Russia

Nonlinear phenomena in polariton topological insulators
YAROSLAV KARTASHOV
Institute of Spectroscopy, Russian Academy of Sciences, Moscow, Russia

Superdiffusive Burgers-Taylor's kinks and nonlinear dispersive shock waves
JUAN I. RAMOS
Universidad de Málaga, Spain

From 2D to 3D in LiCN: A brief history of Nonlinear Dynamics of a Hamiltonian Molecular System
JUAN CARLOS LOSADA
Universidad Politécnica de Madrid, Spain

Divulgative Talk

Rectorado de la Universidad de Málaga
Avda. Cervantes, 2 (downtown)

¿Es reversible el cambio climático antropogénico? Impacto del "agujero de ozono" en el clima del hemisferio Sur
MANUEL PULIDO
University of Reading, United Kingdom

Communications in order of presentation

Escuela de Ingenierías Industriales, Sala de Grados A
Universidad de Málaga (Ampliación del Campus de Teatinos)

On a model for internal waves in a rotating fluid

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ABSTRACT

The purpose of this talk is to bring up some aspects on a model for the propagation of internal waves in a rotating fluid. This is based on incorporating rotational effects in the problem for internal waves described by the Benjamin equation, [2, 3, 1], or, alternatively, on including capillary forces in the Ostrovsky equation, [4, 5], for wave dynamics in a rotating fluid. Preliminary results concerning the regime of validity, the well-posedness of the initial-value problem and the existence of solitary-wave solutions will be presented.

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Center cyclicity on center manifolds of polynomial families of vector fields

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ABSTRACT

We consider polynomial families of ordinary differential equations on $\mathbb{R}^3$, parametrized by the admissible coefficients, for which there is an isolated singularity at which the linear part of the system has one non-zero real and two purely imaginary eigenvalues. We derive theorems that bound the maximum number of limit cycles within the center manifold that can bifurcate, under arbitrarily small perturbation of the coefficients, from any center at the singularity. The bounds are global in that they apply to the system corresponding to any point on any irreducible component of the center variety in the space of parameters. We also derive theorems for such bounds when attention is confined to a single irreducible component of the center variety. Finally, we apply the results obtained to get the cyclicity of the center located at the origin of the well-known Lorenz system. Moreover, we extend the results obtained in [2] about the center cyclicity in the Moon-Rand family. The talk is based on the work [1].

References

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Leaky Surface Wave Propagation for Mass Loading Biosensors: A Numerical Study

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ABSTRACT

Surface acoustic wave (SAW) devices are a promising candidate for Point-of-Care (PoC) testing [1]. These devices relate the mass loading of an analyte to a measurable shift in elastic wave properties. The properties are poorly understood outside of standard configurations such as ST-cut Quartz. Presented is the numerical study into the existence of plane waves in multilayered anisotropic structures, developed based on the Stroh formulation [2]. The effect of dissimilar anisotropies on the existence of leaky and non-leaky SAW modes is explored in a layered half-space model. A second model with an additional layer of viscoelastic material is used to study these waves with change in concentration of analyte. The slowness surfaces are presented for both cases; these are required for designing non-circular annular interdigital transducers [3] for novel circular resonators [4]. The shift in frequency due to changes in analyte concentration is investigated for particular propagation angles and velocity ranges. These cases are at transition states in which the propagating bulk waves allow leaky modes to exist. The differing symmetries of the anisotropic layers allow these leaky modes to come into existance at particular directions governed by the combination of bulk wave solutions between each layer. The structure of these waves suggest a mode of operation based on the behaviour at transition with the possible creation of discontinuities.

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Bautin-type analysis in 1-dimensional averaging theory for periodic differential equations

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ABSTRACT

The classical averaging theory is a tool to study the dynamics of the periodic nonlinear differential systems, see [2]. We use averaging theory for computing T-periodic solutions of λ-families of analytic T-periodic ordinary differential equations in standard form defined on $\mathbb{R}$ analyzing the displacement functions $d(z, \lambda, \varepsilon)$ defined by these equations. Recall that $d : \Omega \times \mathbb{R}^p \times I \to \Omega$ is a real analytic functions, Ω is a bounded open subset of $\mathbb{R}$, $I \subset \mathbb{R}$ is an interval containing the origin, $\lambda \in \mathbb{R}^p$ are parameters, and ε is a small parameter.
We study the branching of the zero-set of d when the parameter ε varies following a Bautin-type analysis [1]. We call the coefficients in the Taylor expansion of $d(z, \lambda, \varepsilon)$ in powers of ε the averaged functions. The main contribution consists in analyzing the role that have the multiple zeros $z_0 \in \Omega$ of the first non-zero averaged function. The outcome is that we bound the maximum number of branches of isolated zeros that can bifurcate from each multiple zero z_0 in two different ways depending on whether (z_0, λ) belongs or not to the analytic set defined by the real variety associated to the ideal generated by the averaged functions in the Noetheriang ring of all the real analytic functions at (z_0, λ). Several examples illustrate our results and they are compared with the classical theory, branching theory and also under the light of singularity theory of smooth maps. The examples range from polynomial vector fields to Abel differential equations and perturbed linear centers. The talk is based on the work [3].

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Periodic solutions for an scalar differential equation modelling optical conveyor belts

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ABSTRACT

In this work, we study the existence of periodic solutions for a differential equation modelling an optical conveyor belt , where the gradient forces that act on the z-axially confined particles are obtained by means of the superposition of two temporally dephased counter propagating complex electromagnetic fields. The dynamic of particles in an axial optical conveyor belt can be modeled by the differential equation:

$$z' = F_z(t, z), \qquad (1)$$

where

$$F_z(t, z) = \frac{\partial V(t, z)}{\partial z},$$
$$V(t, z) = F_0\, f(z) \cos\left(kz - \frac{bt}{2}\right)^2,$$

with $F_0, k, b > 0$.
Using a general theorem from [1] on existence of solutions for boundary-value problems we obtain the existence of periodic solutions for the nonautonomous equation (1) in the following particular cases:

- *Lorentzian axial region strength:* $f(z) = \frac{1}{1+(z/z_0)^2},\ z_0 > 0.$
- *Gaussian axial region strength* $f(z) = exp(-2\frac{z^2}{z_0^2}),\ z_0 > 0.$

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Information flow in a two-scale stochastic dynamical system using ordinal symbolic analysis

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ABSTRACT

Although the resolved dynamical equations of atmospheric or oceanic global numerical models are well established, the development and evaluation of parameterizations that represent subgrid-scale effects pose a big challenge [1]. Small-scale waves in the geophysical systems generate in a region, then propagate conservatively long distances to finally break and deposit their momentum far away from their generation location. Therefore, the system is characterized by strong nonlinear and remote interactions between the different scales. To mimic these interactions between subgrid-scale and large-scale dynamics, we use a proof-of-concept two-scale stochastic dynamical system based on Lorenz 1996 with long-range interactions between the two-scales. In this work, time series obtained from these systems are evaluated with information flow measures, i.e. mutual information and transfer entropy[2]. The measures are obtained via ordinal symbolic analysis using the Bandt-Pompe[3] symbolic data reduction in the signals. By comparing different experiments, we show that even when the dynamics of the large-scale variables of the systems has a short-range correlations, small-scale variable may give place to long-range information flow. We evaluate the potencial of information measures as a tool for establishing the structure and functional dependencies to be represented by the subgrid-scale parameterizations. .

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Multistability in the dynamics of bubble contrast agents

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ABSTRACT

In this talk we consider dynamical systems for the description of the oscillations of spherical gas bubbles in a liquid. Such systems can be used for modelling of ultrasound contrast agents, which are used for enhancing ultrasound visualization [1]. While there are a lot of different models of microbubble contrast agents and it is known that complex dynamics may occur in such models (see, e.g. [2, 3, 4]), only few of them were thoroughly studied from a dynamical systems point of view. In addition, the knowledge of the location in the parameter and initial conditions space of regions of chaotic and regular dynamics is important for applications since both types of the dynamics may be either beneficial or undesirable depending on a particular usage of microbubbles. Therefore, understanding of the nonlinear dynamics of microbubbles is an important problem.

Here we investigate two models of encapsulated gas bubbles dynamics. The first one takes into account the liquid viscosity and compressibility and bubble's shell. The shell term is chosen according to the de-Jong model [5]. In the second one we in addition take into consideration impact of an elastic wall near which a bubble oscillates. We show that in both of these models complex dynamics can occur. For example, there is a possibility of coexistence of two periodic attractors or periodic and chaotic ones. We study the dependence of dynamics type on the two control parameters that are magnitude and frequency of the external pressure field. We find regions of the parameters which correspond to regular, chaotic and multistable dynamics. To this aim we use several approaches, including the numerical continuation and the perpetual points methods (see, e.g. [6, 7]).

This work is supported by Russian Science Foundation, grant number 17-71-10241.

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Robustness of dynamically gradient multivalued dynamical systems

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ABSTRACT

In this work we show under suitable assumptions that a dynamically gradient multivalued semiflow is stable under perturbations. These results extend previous ones in the single-valued framework to the case where uniqueness of solution does not hold.
Afterwards, we consider a Chafee-Infante problem, where the equivalence of weak and strong solutions and the set of fixed points is analyzed. Also, the weak solutions of a particular family of Chafee-Infante problems approximating a differential inclusion studied in [1] generate a dynamically gradient multivalued semiflow with respect to suitable Morse sets. Moreover, we check that this family of Chafee-Infante equations verifies the hypotheses of the robustness theorem.
In a more general context, describing the structure of global attractors is an open question on which focus efforts.

Acknowledgements

This research was supported by the following projects: R. Caballero is a fellow of Programa de FPU del MECD, reference FPU15/03080; P. Marín-Rubio was supported by projects PHB2010-0002-PC (MECD-DGPU), MTM2015-63723-P (MINECO/FEDER, EU) and P12-FQM-1492 (Junta de Andalucía); and J.Valero by projects MTM2015-63723-P, MTM2016-74921-P (MINECO/FEDER, EU) and P12-FQM-1492 (Junta de Andalucía).

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Global connections near a triple-zero singularity

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ABSTRACT

In this work, we consider an unfolding of a normal form of the Lorenz system [1] near a triple-zero singularity

$$\left\{\begin{array}{rcl} \dot{x} & = & y, \\ \dot{y} & = & \epsilon_1 x + \epsilon_2 y + xz + Byz, \\ \dot{z} & = & \epsilon_3 z + x^2 + Dz^2, \end{array}\right. \quad (1)$$

that appears when $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0$. From this codimension-three point a double-zero bifurcation emerges when $\epsilon_1 = \epsilon_3 = 0$ and $\epsilon_2 \neq 0$.

The analysis of this unfolding provides partial results that can be extended by means of the adequate numerical continuation methods [2]. Specifically, it is demonstrated that a curve of heteroclinic connections (joining the equilibria located on the z-axis) emerges from this double-zero bifurcation. Moreover, this curve has a degenerate point from which infinitely many homoclinic connections (related to one of these equilibria) emerge.

In this way, we can partially understand the dynamics in the vicinity of the triple-zero singularity exhibited by system (1).

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Computational methods for compactons in the Rosenau–Hyman equation

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ABSTRACT

Compactons are solitary waves of compact support of evolution equations with nonlinear dispersion characterized by robustness under mutual interactions [1]. The lack of analytical techniques to study the interaction of these solitary waves requires the use of numerical methods. Padé approximations with artificial viscosity have been used with success for the Rosenau–Hyman equation [2], but they introduce artificial trailing tails into the solution; they can be removed by properly designing the artificial viscosity [3]. This computational techniques can be applied to other evolution equations with compacton solutions [4]. In this oral communication their current state of the art will be reviewed.

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On the stability of positive periodic solutions in MEMS via lower and upper solutions

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ABSTRACT

This talk considers the existence and linear stability of positive periodic solutions for the Nathanson's model and the Comb-drive model of Micro-Electro-Mechanical-Systems (MEMS). Our results contribute to a better understanding of these models and therefore, they add up to the set of tools to decide the range of usefulness in sensing and acting of these two models. Both models are considered without damping and having an AC-DC input voltage $\mathcal{V}(t) > 0$ of period $T^* > 0$. For the Nathanson's model, we prove the existence of two T^*-periodic solutions $0 < \varphi(t) < \psi(t)$, under the assumption that the voltage is less than the pull-in voltage and greater than a certain positive constant depending on the temporal frequency of $\mathcal{V}$ and physical parameters. The solution φ is elliptic and, if the parameter $\lambda := \frac{min \mathcal{V}}{max \mathcal{V}}$ is greater than a certain critical value, the solution ψ is hyperbolic. We point out that if $\mathcal{V}$ is greater than the pull-in voltage, there are not any T^*-periodic solutions. Under the same conditions that guarantee the existence, we prove the uniqueness of these two solutions and we classify their linear stability. We can view this existence/uniqueness/linear stability classification theorem as an extension of the known results for the autonomous case when the voltage $\mathcal{V}$ is constant and as an extension of results in [1] for the non-damping case. The Comb-drive model is studied under a cubic stiffness with coefficient $\alpha > 0$, and its dynamic depends on whether $\alpha \leq 2$ or $\alpha > 2$. In any case, since the trivial equilibrium (the zero solution) is elliptic for an adequate range of $\mathcal{V}$, then it isolates this solution suggesting the existence of positive periodic solutions. We will prove the existence of periodic positive solution. When $0 < \alpha \leq 2$, this periodic solution is unstable for λ greater than a certain critical value and $\mathcal{V}$ below the pull in voltage. For the case $\alpha > 2$, similar to the Nathanson case, we prove the existence of two positive periodic solutions one of which is linearized stable and the other is unstable. The methodology uses the Lower and Upper Solution Method. Some numerical examples are provided to illustrate the results.

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Finite diference methods based on time-linearization for the GRLW equation

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ABSTRACT

The generalized regularized long wave (GRLW) equation is used to model unidirectional wave propagation in fluid dynamics, turbulence, acoustics, plasma dynamics, etc., and has been studied by different authors with techniques that include Petrov-Galerkin methods [1], finite difference [2, 3, 4] and finite element methods [5], and can be written as

$$u_t + \alpha u_x + \epsilon \left(u^p\right)_x = \mu u_{xx} + \delta u_{txx}, \qquad -\infty < x < \infty, \qquad t > 0, \qquad (1)$$

where u, t and x denote the (nondimensional) amplitude of the wave, time and spatial coordinate, respectively, and $\alpha \geq 0$, $\epsilon \geq 0$, $\mu \geq 0$, $\delta \geq 0$, and $p \geq 2$ are nondimensional parameters that are associated with linear and nonlinear advection, viscosity, dispersion, and nonlinearity, respectively. For μ=0, the above equation has a solitary wave solution and three conserved quantities (mass, momentum and energy) [6].

In this paper, the effects of both the initial conditions and the values of the parameters on the propagation of solitary waves are assessed by considering initial conditions corresponding to the analytical solution of Eq. (1) with μ=0, and Gaussian and triangular functions. The equation is solved numerically by means of several finite difference methods based on time-linearization techniques.

For the inviscid and viscous GRLW equations with bell-shaped initial conditions, the results show an initial adjustment of the solution whereby the leading part undergoes steepening, increases its amplitude and may break up from the trailing part. When a balance between non-linearity and dispersion is reached, a solitary wave emerges from the break-up and propagates along a straight line towards the downstream boundary in the absence of viscosity. In the presence of viscosity, the wave trajectory is curved and the wave amplitude decreases with time. For larger viscosities, solitary waves may not form. Instead, a damped wave with a steep front whose slope decreases with time is observed. The number and speed of the solitary waves that are formed after breakup increase as the dispersion parameter decreases. The effect of the linear advective term is to push the wave toward the downstream boundary. The number of solitary waves increases and the distance between solitary waves decreases as the width of the initial conditions increases regardless of their shape.

Keywords: GRLW equations, finite difference methods, linearization, solitary waves

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Time-diffracting beams in self-focusing Kerr media: self-trapped propagation below the critical power

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ABSTRACT

Time-diffracting beams are a new class of spatiotemporal wave packets whose diffraction-free behavior is based on spatiotemporal coupling effects, as recently described and demonstrated experimentally [1, 2, 3, 4]. Their temporal-transversal structure reproduces the axial-transversal structure of monochromatic light beams, i. e., diffraction takes place in time and not along the longitudinal direction [2, 3].

We have studied the nonlinear propagation of time-diffracting beams in self-focusing (cubic) Kerr media. Although stable, two-dimensional localized structures (spatial solitons) are not supported by cubic media, time-diffracting beams are demonstrated to reshape spontaneously into a propagation-invariant, seemingly robust, nonlinear wave packet of finite power. Below the critical power for collapse, the spatiotemporal structure of the attracting wave packet is that of the self-focusing of the standard monochromatic beam of the same shape and power, i. e., self-focusing is swapped from the axial direction to time. Above the critical power (and with a suitable mechanism halting collapse), the spatiotemporal structure is also that of the halted collapse of the standard monochromatic beam.

The well-known, unstable Townes beam, and other spatial "solitons", represent the limit of these wave packets when localization in time is lost. These findings may open new perspectives in the longstanding problem of synthesizing truly robust, localized waves in media with the ubiquitous Kerr nonlinearity.

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A necessary condition to have Weierstrass integrability

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ABSTRACT

In this talk we give a necessary condition to have Weierstrass integrability. We use the solutions $y = f(x)$ of the differential system in the plane and its associated Weierstrass cofactors. The necessary condition can be used to detect polynomial differential systems which are not Liouville integrable.

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Analytic integrability of planar systems with non null linear part

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Abstract

Recently, in [2] solved the analytic integrability problem around a nilpotent singularity for differential systems in the plane under generic conditions. In this work we solve the remaining case completing the analytic integrability problem for such singularity.

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Suppressing chaos in damped driven systems by incommensurate excitations

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ABSTRACT

The possibility of suppressing chaos in dissipative nonautonomous systems by additional incommensurate chaos-suppressing excitations is theoretically demonstrated through the universal example of a perturbed Duffing oscillator by considering rational approximations to the irrational ratio $\frac{\Omega}{w}$ between the chaos-suppressing and chaos-inducing frequencies. For each chosen rational approximation, analytical predictions for the suitable amplitudes and initial phases of the chaos-suppressing excitation are numerically confirmed for small amplitudes and at the predicted suitable initial phases. For the significant case $\frac{\Omega}{w} = \Phi$ (golden section), we study the structural stability of the suppression scenario as the respective convergents approximate the corresponding irrational ratio. Our theory predicts and numerical simulations confirm that the values of the suitable amplitudes are rather insensitive to high order convergents, while the number and values of the suitable initial phases critically depend on each particular convergent according to the maximum survival of a relevant spatio-temporal symmetry.

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The role of discrete breathers in establishing the Fourier heat transport in ϕ^4 chain

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ABSTRACT

Anomalous (non-Fourier) heat transport is no longer just a theoretical issue since it has been observed experimentally in a number of low-dimensional nanomaterials, such as SiGe nanowires, carbon nanotubes, and others. To understand these anomalous behaviors, exploring the microscopic origin of normal (Fourier) heat transport is a fascinating theoretical topic. However, this issue has not yet been fully understood even for one-dimensional (1D) model chains, in spite of a great amount of thorough studies done to date. From those studies, it has been widely accepted that the conservation of momentum is a key ingredient to induce anomalous heat transport, while momentum-nonconserving systems usually support normal heat transport where Fourier's lawis valid.But if the nonconservation of momentum is the reason, what is the underlying microscopicmechanism for the observed normal heat transport? Here we carefully revisit a typical 1D momentum-nonconserving ϕ^4 model, and we present evidence that the mobile discrete breathers, or, in other words, the moving intrinsic localized modes with frequency components above the linear phonon band, can be responsible for that. Main results have been published in [1].

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Orbital–Reversibility for vector fields. Application to center problem.

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ABSTRACT

We consider an analytic n-dimensional autonomous system of differential equations given by:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}),\ \mathbf{x} = (x_1, \ldots, x_n)^T \in \mathbb{R}^n, \tag{1}$$

having an isolated equilibrium point located at the origin. We study the orbital-reversibility problem, which consists in determining if it admits some reversibility (possibly nonlinear) modulo $\mathcal{C}^\infty$-equivalence (see [3], [5], [6]). The problem of determining if system (1) has some reversibility is considered in [1], [2], [3] and [5]. In this work, we determine if there exists a time-reparametrization such that the transformed system admits some reversibility. We determine the existence of a normal form modulo reversible terms, that is, we simplify as much as possible the non-reversible terms. This allows us to give necessary conditions for the orbital-reversibility of system (1). Taking into account these conditions, we formulate a suitable algorithm to detect orbital-reversibility. We apply the results obtained to the center problem for planar vector fields. More concretely, to planar nilpotent vector fields, see [4].

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Generalising the BDS test to check i.i.d. in high-frequency financial time series

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April 13, 2018

ABSTRACT

The statistical analysis of financial time series usually have considered time series sampled at fixed time intervals, mainly daily data. Recently there are some publications about models that can fit high-frequency data, even tick-by-tick. Particularly, we focus on high-frequency-based volatility models, HEAVY models [1]. Our purpose would be generalise the use of the well-known BDS test as a residual diagnostic tool of this kind of models. Brock *et. al.,* [2] proposed the BDS statistical test as a nonparametric method for testing serial independence and identically distributed hypothesis based on the correlation integral [3] that involves reconstructing the state space from a single time serie using Taken's theorem [4]. So far this reconstruction method was apply considering only time series sampled at fixed time intervals. The reconstruction of the attractors using high-frequency time series with a variable time-lapse between each observation need several considerations [5]. In this paper, we generalise the BDS as a nonlinear test considering tick-by-tick financial time series. Additionally, we check i.i.d. hypothesis in the residuals of HEAVY models as a diagnostic tool for this kind of nonlinearity modeling.

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Punchets: nonlinear transport in Hamiltonian pump-ratchet hybrids

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ABSTRACT

"Punchets" are hybrids between ratchets and pumps, intended as models of metal or semiconductor surfaces irradiated by a collimated laser beam [1]. They combine a spatially periodic static potential, typically asymmetric under space inversion, with a local driving that breaks time-reversal invariance. Their crucial feature is irregular driven scattering between asymptotic regions supporting periodic (as opposed to ballistic) motion. With all binary spatio-temporal symmetries broken, scattering in punchets generally induces directed currents. We here study the underlying nonlinear transport mechanisms, from chaotic scattering on the level of individual trajectories to the parameter dependence of currents evaluated as ensemble averages, in three types of Hamiltonian models, (i) with spatially periodic potentials where only in the region of driven scattering, spatial and temporal symmetries are broken, and (ii), spatially asymmetric (ratchet) potentials with a driving that only breaks time-reversal invariance. As a more realistic model of laser-irradiated surfaces, we consider (iii), a driving in the form of a running wave confined to a compact region by a static envelope. In this case, the induced current can even run against the direction of wave propagation, drastically evidencing of its nonlinear nature. Quantizing punchets is indicated as a viable research perspective.

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Padé numerical schemes for the sine-Gordon equation

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ABSTRACT

The sine-Gordon equation

$$u_{tt} - u_{xx} + \sin u = 0,$$

where the subscripts denote partial differentiation, arises in many branches of mathematical physics [1]. In this work, this equation is solved by means of four second–order time accurate finite difference methods that use Padé approximations in space [2], [3]. The accuracy and stability of the numerical solution is studied by means of the analysis dicretization error and invariants for both kink–antikink and breather solutions of the sine-Gordon equation as functions of the time steps and spatial grid sizes. The new four Padé methods are compared among them and with the most widely used scheme for the sine-Gordon equation, the Strauss-Vazquez conservative method [4]. The good accuracy and stability of the Padé methods are shown after an extensive set of simulations. In terms of computational cost, the most accurate and efficient Padé scheme is the eighth-order one, as expected since the solutions of the sine-Gordon equation are smooth and the same time-integration scheme is used for all the methods.

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Localized vibrational modes in metals and alloys

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ABSTRACT

Intrinsic localized modes or discrete breathers are studied extensively in last three decades in different physical systems. One of the most important features of the mentioned phenomenon is the ability to concentrate considerable amounts of energy in relatively small volumes. This can play a great role in case of crystal lattices because most of condensed matter objects possess crystalline long range order structure with inevitable presence of defects. Each of those has own contribution to structure dynamics and properties. Evolution of defect structure of the material is connected with overcoming of corresponding potential barriers. This process in turn is related to the energy localization in the crystal lattice. Recently it was shown that DBs can exist in pure metals and ordered alloys contributing to the evolution of crystal structure and properties. Excitation of localized modes in crystals can be performed using anzats (initial atomic displacement function) or localization of delocalized nonlinear modes. Effectiveness of different approaches to DB excitations and their possible role in structural evolution in various metals and ordered alloys are discussed.

The research was supported by Russian Scientific Foundation, Grant No 16-12-10175

Long-lived vortex soliton arrays nested in flat-top nonlinear Bessel beams

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ABSTRACT

Ideally, optical vortex solitons need an infinite plane wave background to exist, but in practice they are nested in a broad background beam [1, 2]. Diffraction and self-defocusing broaden the background, making the vortex solitons to decay quickly, and limiting severely the implementation of their promised applications in, for instance, particle trapping, laser material processing, or in vortex-induced waveguides [3]. We report here on nonlinear Bessel beams in the self-defocusing Kerr medium as solitary-like, diffraction-free beams with arbitrarily wide and flat intensity profiles. These properties make these beams the "natural" background beams where the nested vortex solitons are seen to survive for propagation distances that are one order of magnitude larger than in the common Gaussian or super-Gaussian beam backgrounds, and where their interaction dynamics mimics the simple, and hence predictable dynamics of vortex solitons in the uniform background [4]. We have also found that dissipation does not attenuate these flat nonlinear Bessel beams; instead it contributes to their stable propagation against perturbations. This property further prolongs the distance along which the vortex solitons subsist without appreciable decay, and remain therefore useful for their intended applications [5].

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