Solitary waves in the NonLinear Dirac Equation

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In the last decades, the Nonlinear Schrödinger Equation has been the most ubiquitous model for nonlinear waves in optics, atomic physics, fluid mechanics, condensed matter and mathematical physics. However, its relativistic analogue, the Nonlinear Dirac Equation, has been forgotten despite its appearance almost 80 years ago in the context of high-energy physics.

This trend is starting to change because the Nonlinear Dirac Equation is emerging in physical systems of considerable interest, which includes Bose-Einstein condensates in honeycomb lattices, atomically thin 2d Dirac materials such as graphene, silicene or germanene, or honeycomb photorefractive lattices (the so-called photonic graphene).

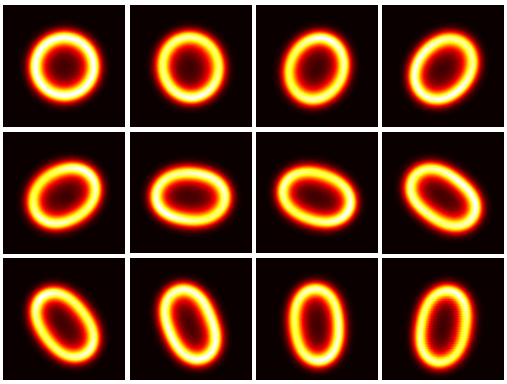

FIGURE 1. Evolution of an unstable soliton in (2+1)D settings.

One of the main interesting features of the Nonlinear Dirac Equation is that it permits the existence of soliton solutions. Mathematical analysis has demonstrated the stability of such solitons in (1+1)D settings. Solitons in (2+2)D systems have prone to oscillatory instabilities for low frequency, leading to deformation and rotation of the soliton (see Fig. 1); vortex solitons are unstable for every frequency and oscillatory instabilities lead to vortex breaking (see Fig. 2).

The aim of the present talk is to present the main models of the Nonlinear Dirac Equation and the most recent results regarding the stability of solitons and vortices in one-, two- and three dimensional settings, as those shown in the paragraph above, and comparing them with the main features of solitons in the relativistic (Nonlinear Schrödinger) limit.

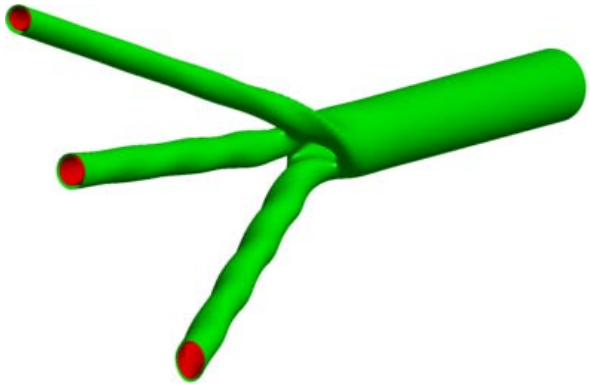

FIGURE 2. Isosurface for the time evolution of the density of an unstable vortex in (2+1)D settings.

Keywords: nonlinear Dirac equation, solitons, vortices.

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