Nonlinear Localized Excitations in Discrete Systems. Electrical lattices

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Dickinson College,
Pennsylvania, USA,
August 30, 2012

Seville

Roman, Jewish, Muslim and Christian city, it became the economic center of the Spanish Empire

Now, Seville is the fourth largest city of Spain

University of Seville

More than 500 years of experience in knowledge management

A great team of more than 70,000 members

http://www.us.es/eng

Nonlinear Physics Group of the University of Seville

http://grupo.us.es/gfnl/

The GFNL is an interdepartamental research group with members from the Physics Faculty (Theoretical Physics) and the Department of Applied Physics I

Francisco Romero Group director

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Jesús Cuevas

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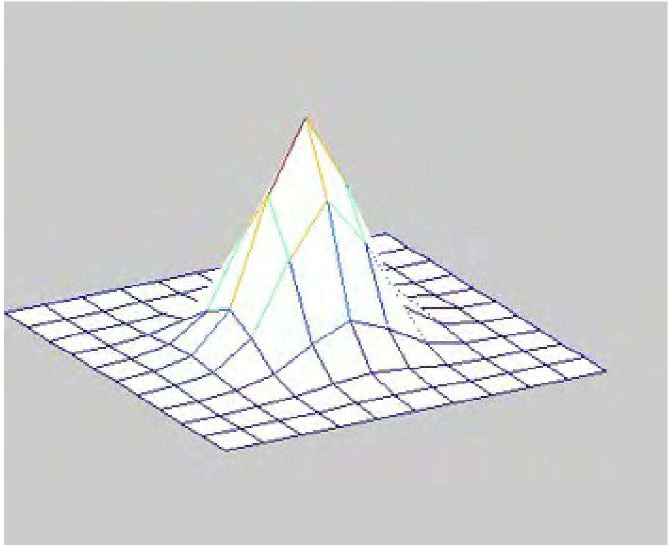
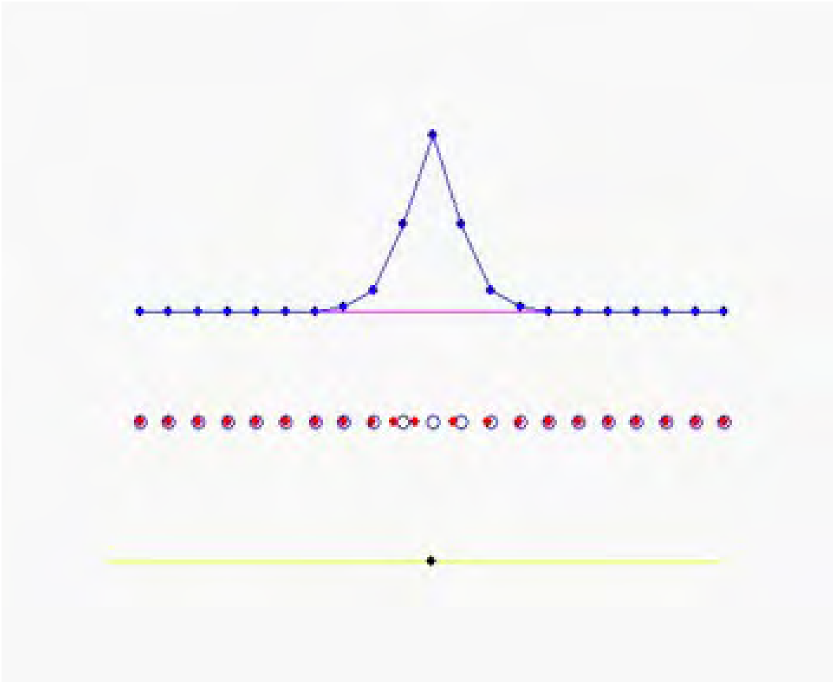
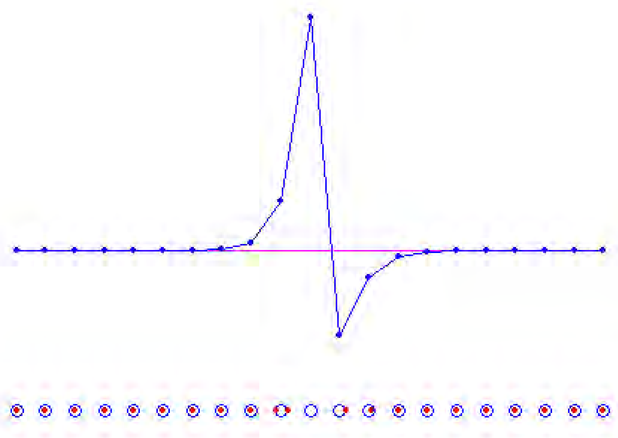
José María Romero

iscrete beathers. Nonlinear localization

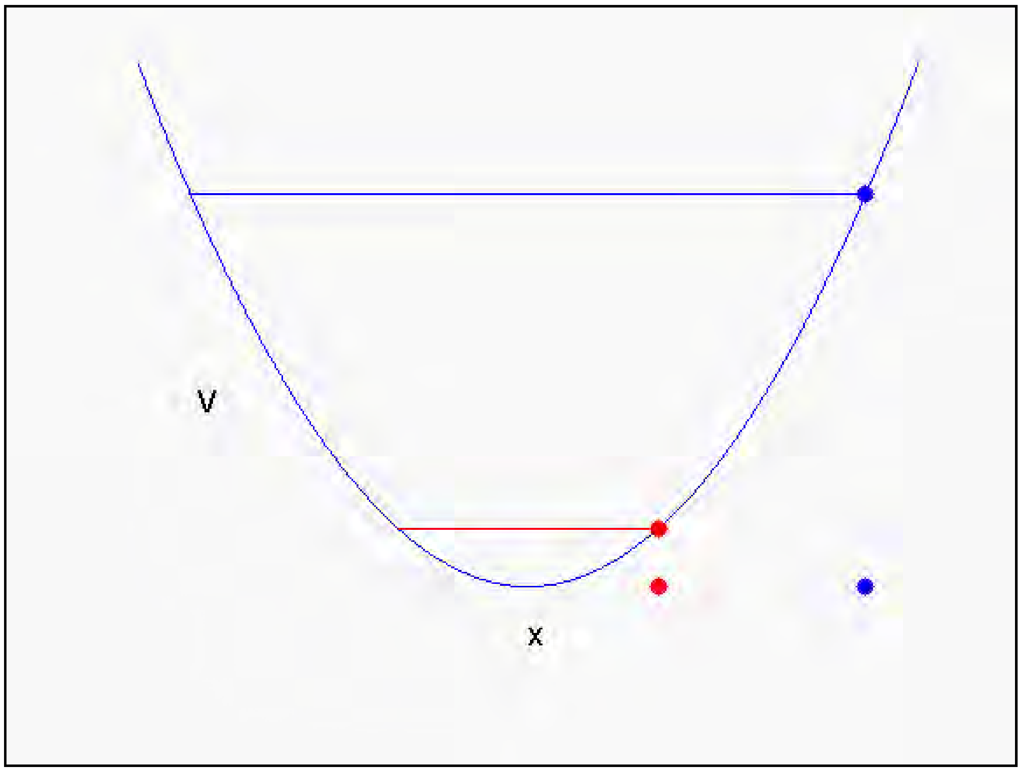
Nonlinear discrete systems.
Vibrational localized states exist. No Anderson localization

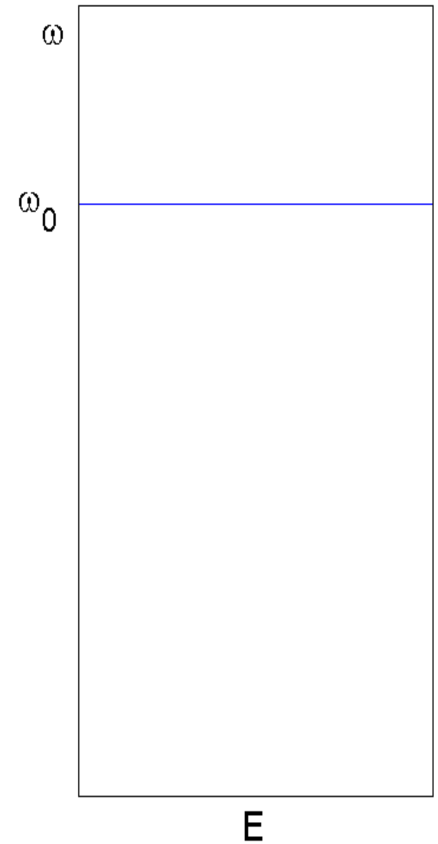




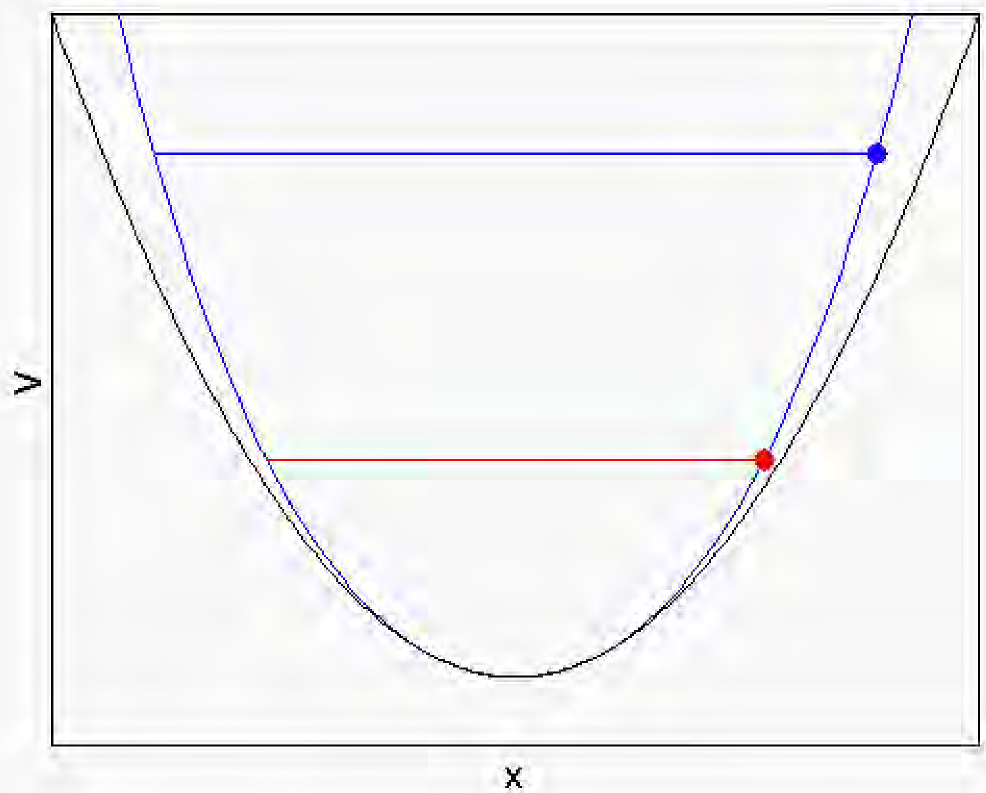


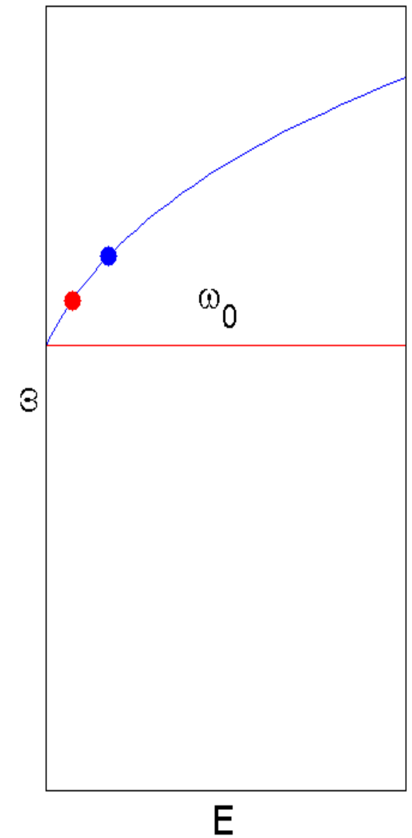
Linear oscillator : $V(x) = \frac{\omega_0^2}{2} x^2$



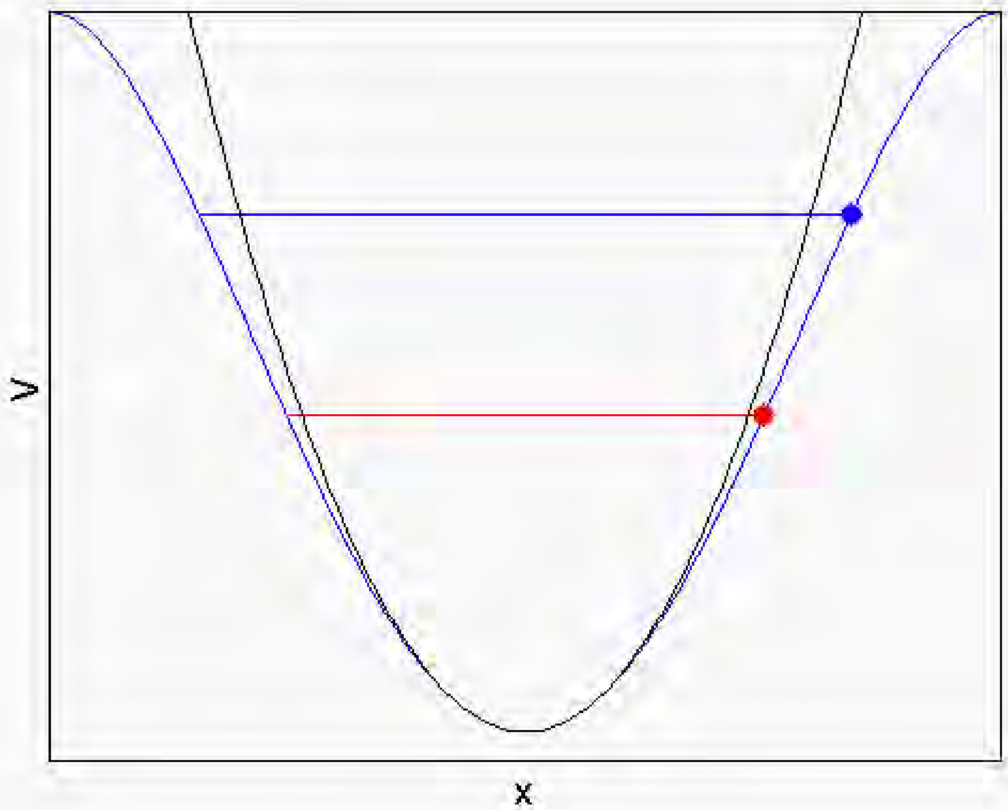


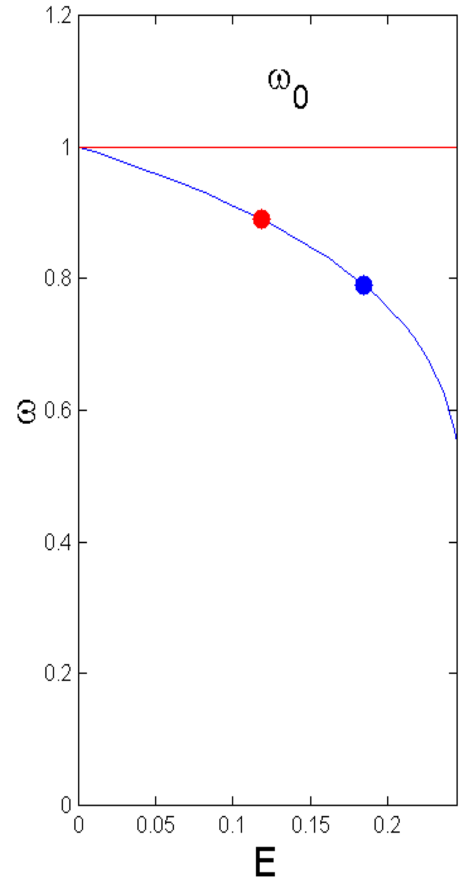
Nonlinear hard oscillator : $V(x) = \frac{\omega_0^2}{2} x^2 + \frac{\epsilon}{4} x^4, \epsilon > 0$





Nonlinear soft oscillator : $V(x) = \frac{\omega_0^2}{2}x^2 - \frac{\epsilon}{4}x^4, \epsilon > 0$





A great number of systems can be described by oscillator networks (crystals, biomolecules ...)

$$H = \sum_n \left[\frac{1}{2} \dot{u}_n^2 + V(u_n) + \sum_m W(u_n, u_m) \right]$$

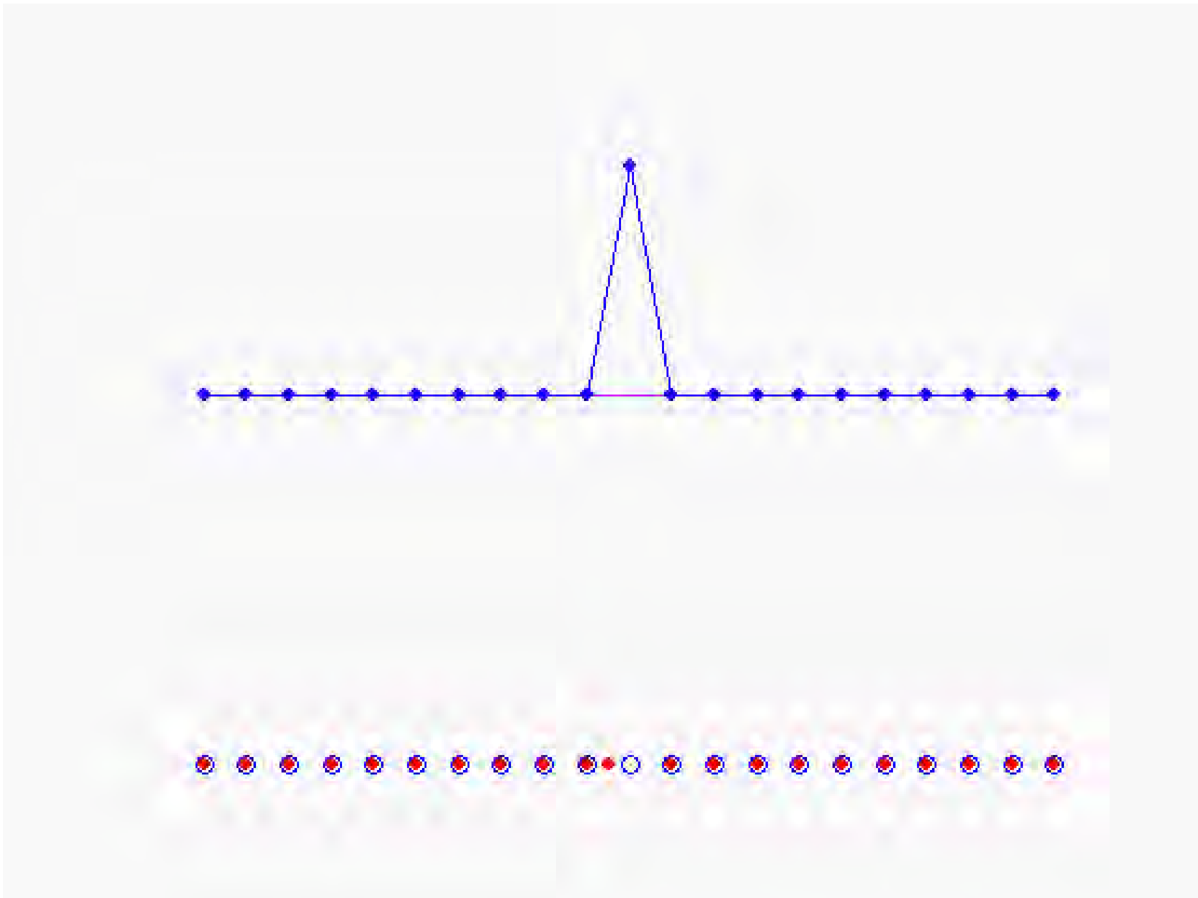
Kinetic Energy

On-site potential

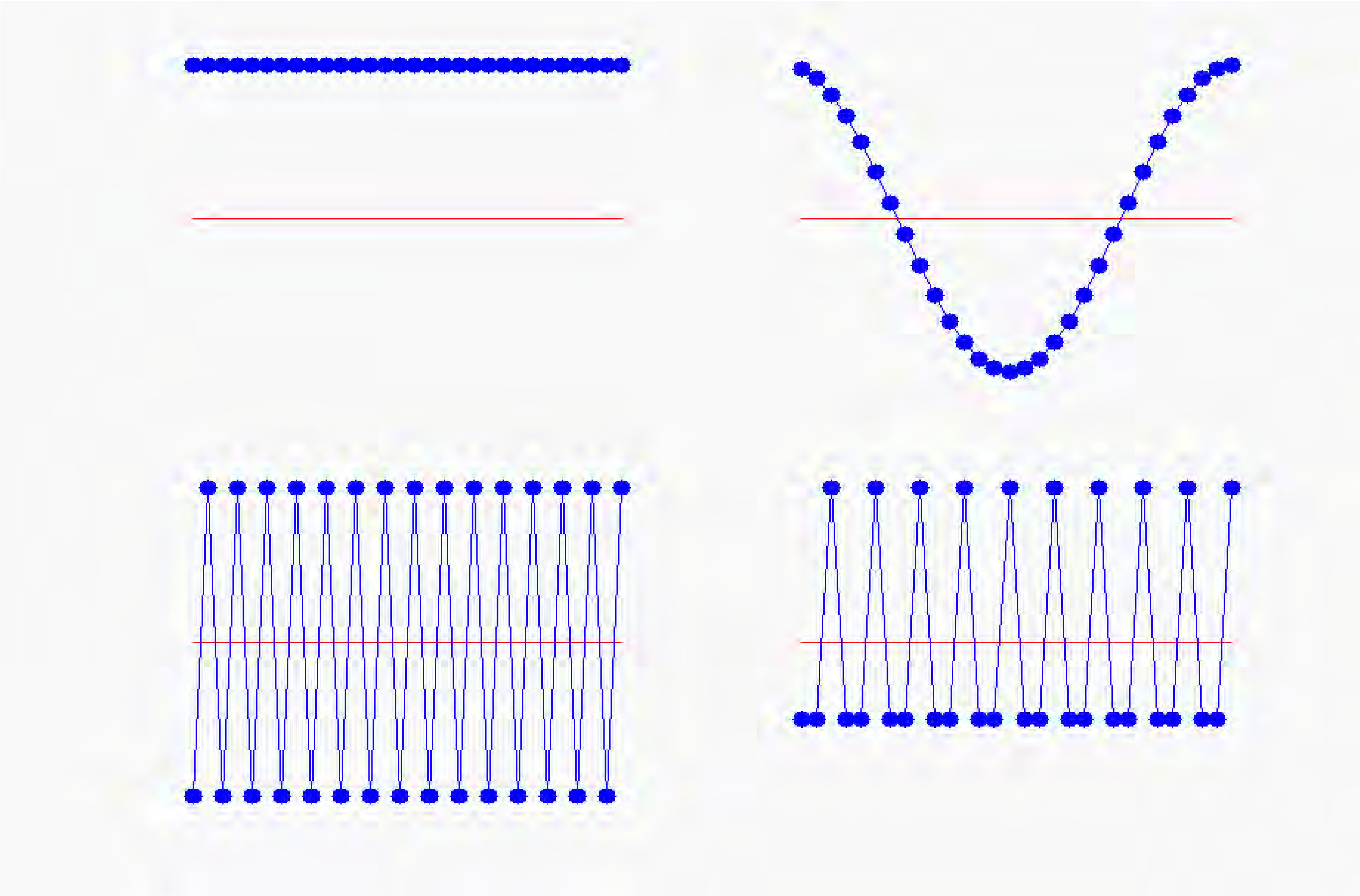
Interaction potential

Higher dimension!

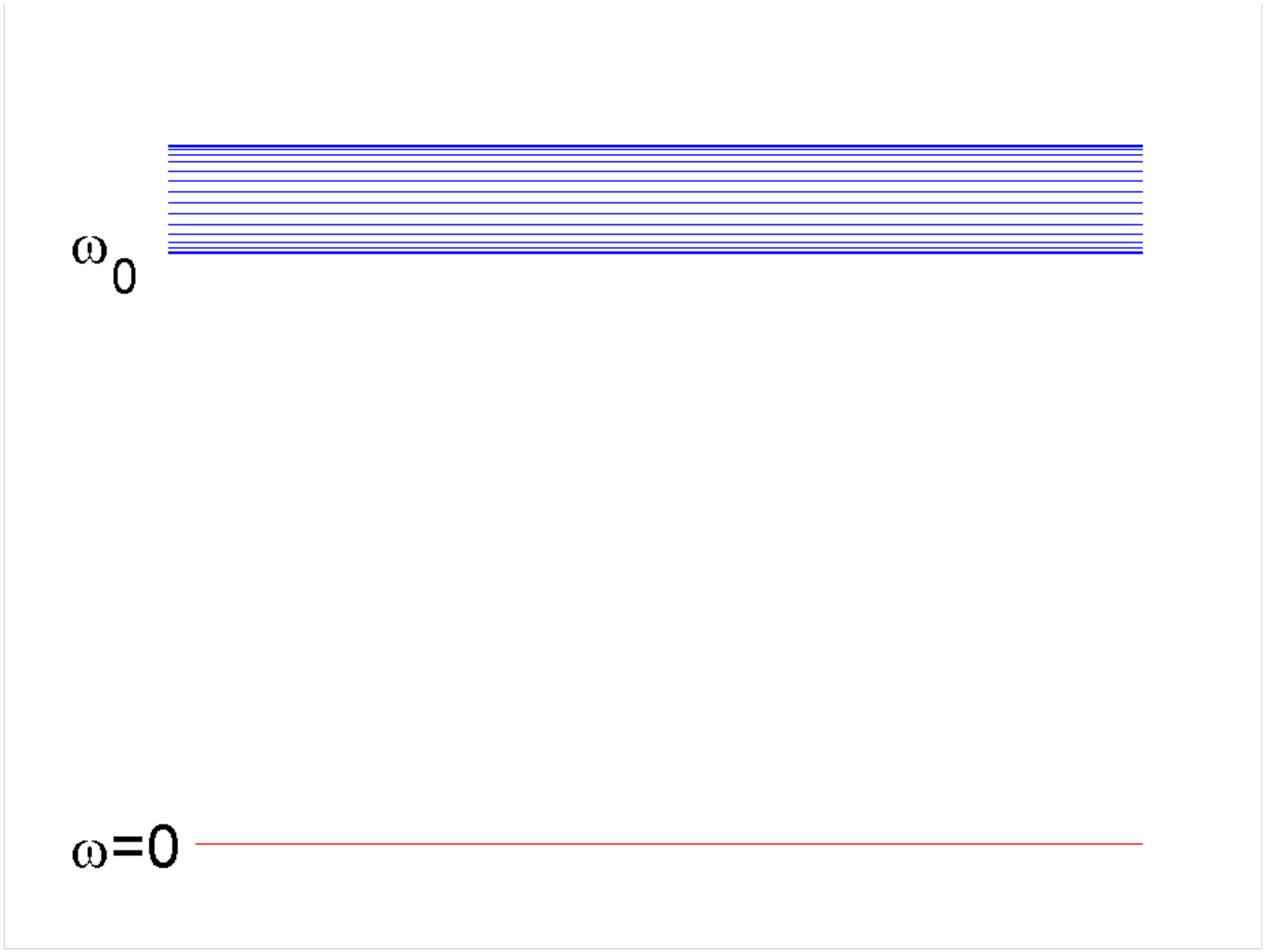
Linear lattices $V = \sum_n \left[\frac{\omega_0^2}{2} u_n^2 + \frac{C}{2}(u_n - u_{n-1})^2\right]$



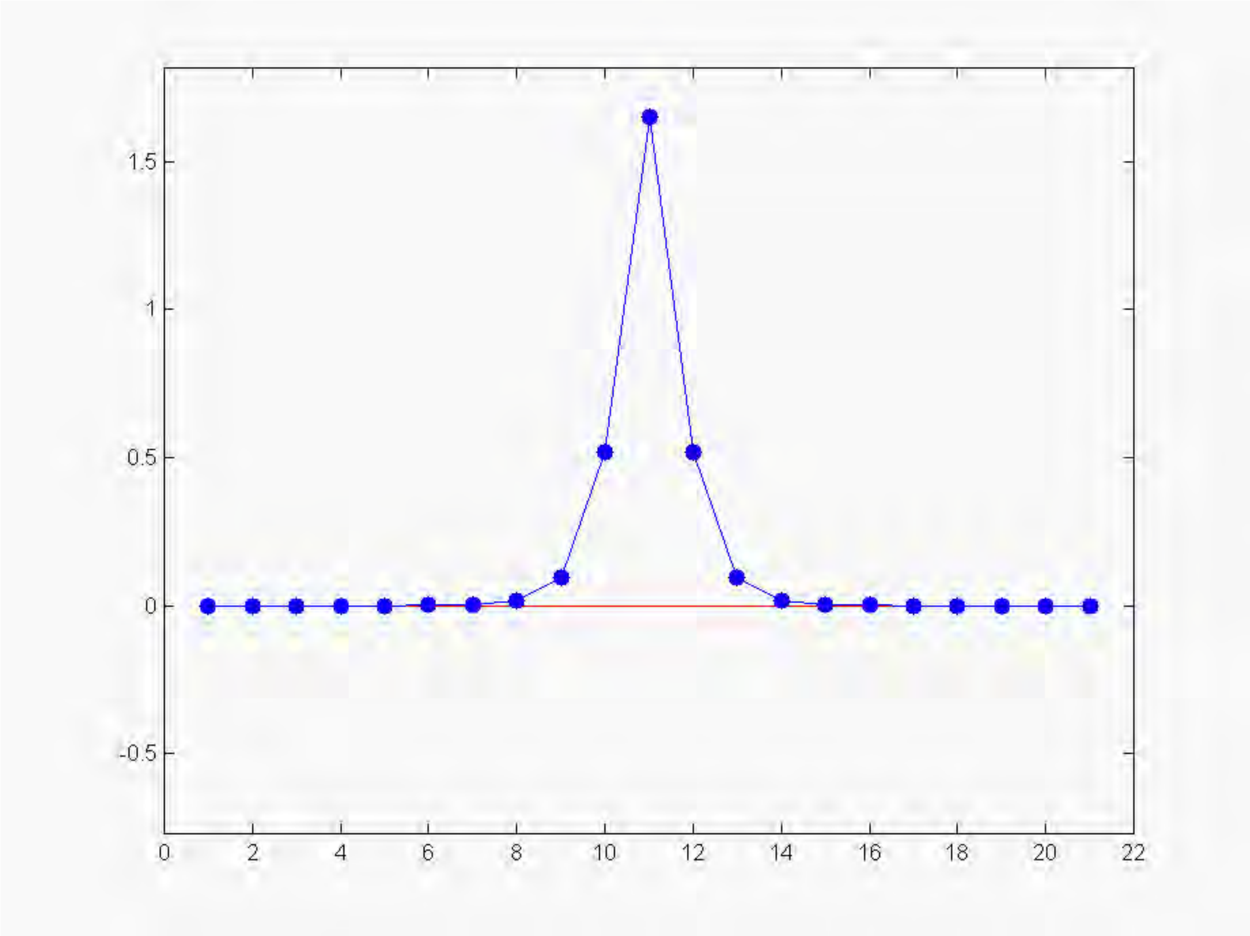
Normal modes!



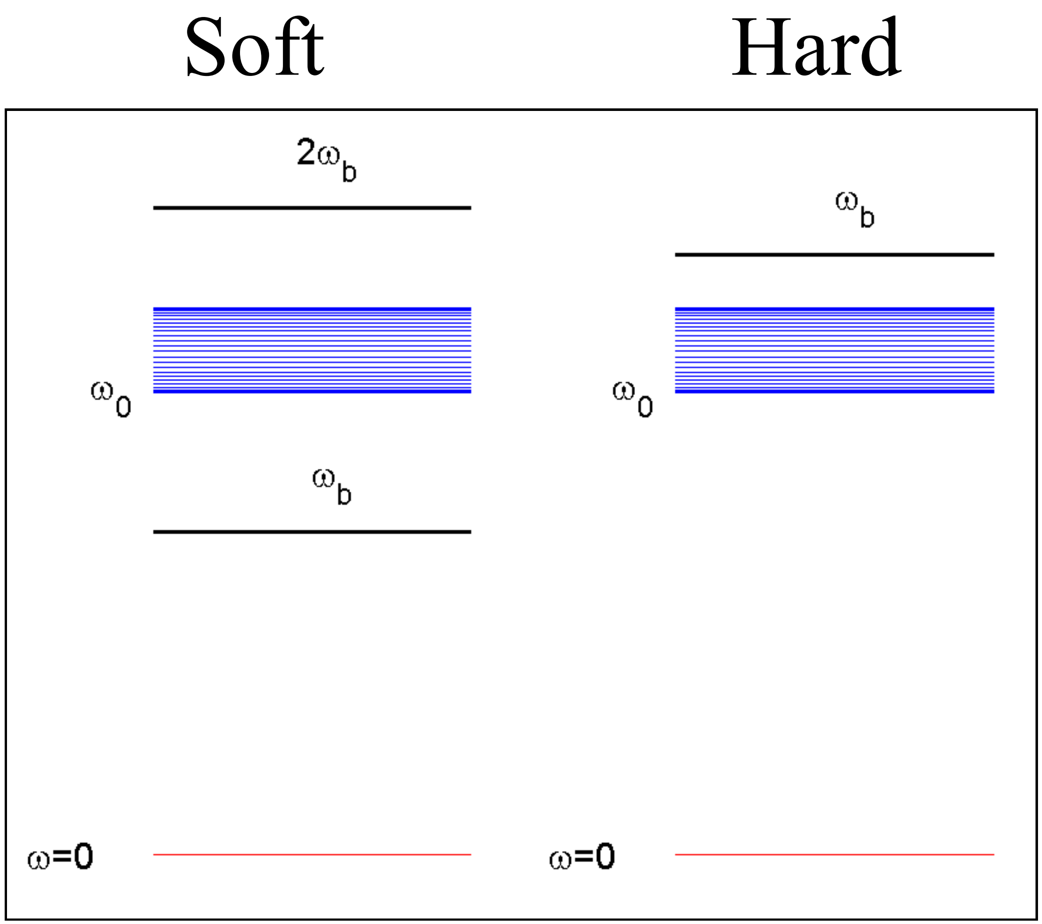
Phonons: $\omega^2{}_{ph} = \omega_0^2 + 4\,C\sin^2 q/2$



Nonlinear network $V = \sum_n \left[V(u_n) + \sum_m W(u_n, u_m) \right]$



stence and stability of discrete breathers (1994)



$$n\,\omega_b \notin [\omega_0, \omega_{f,max}], \omega_b'(E) \neq 0$$

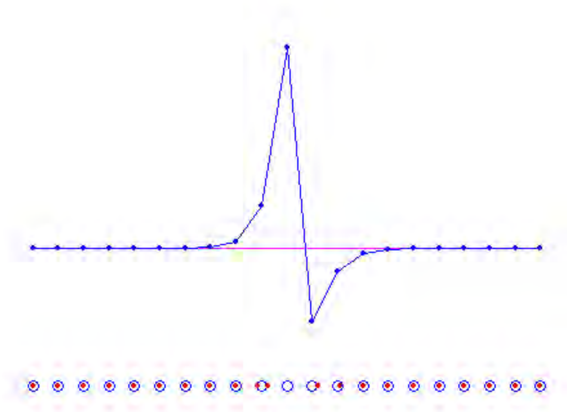
heoretical (and numerical) results

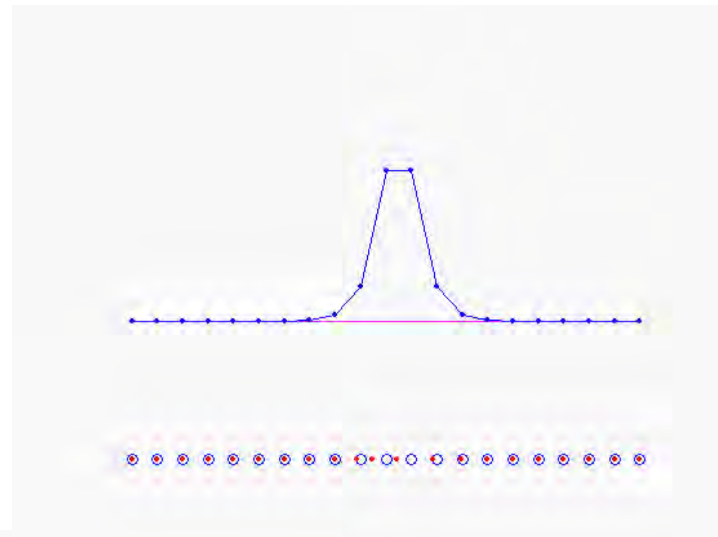
Extension to FPU systems (no on-site potential and nonlinear coupling) and driven-damped networks

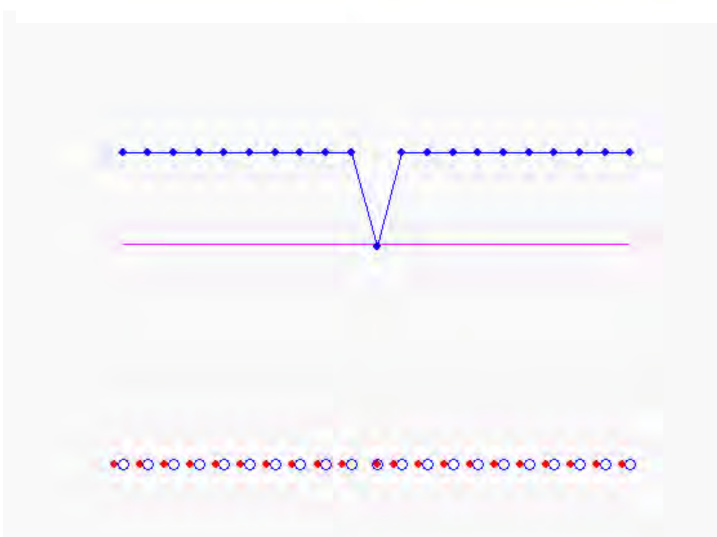
Exact periodic and localized solutions of the dynamical equations that exist due to nonlinearity and discreteness

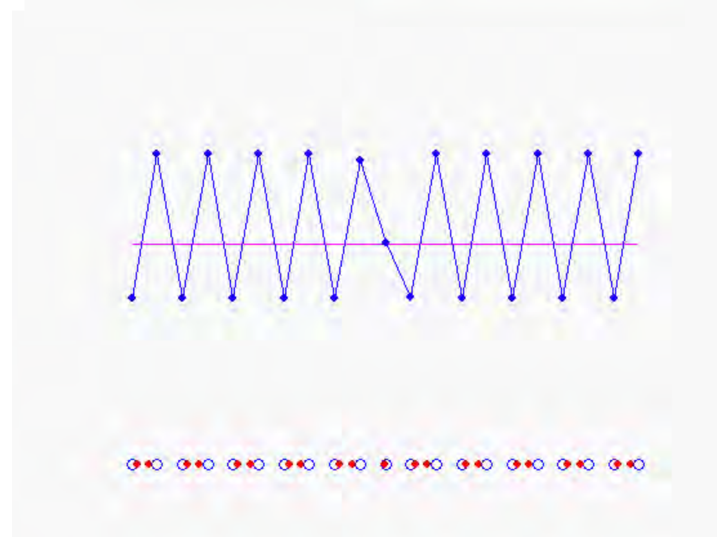
Big amount of theoretical (and numerical) work. General conditions!

Breathers, multibreathers…

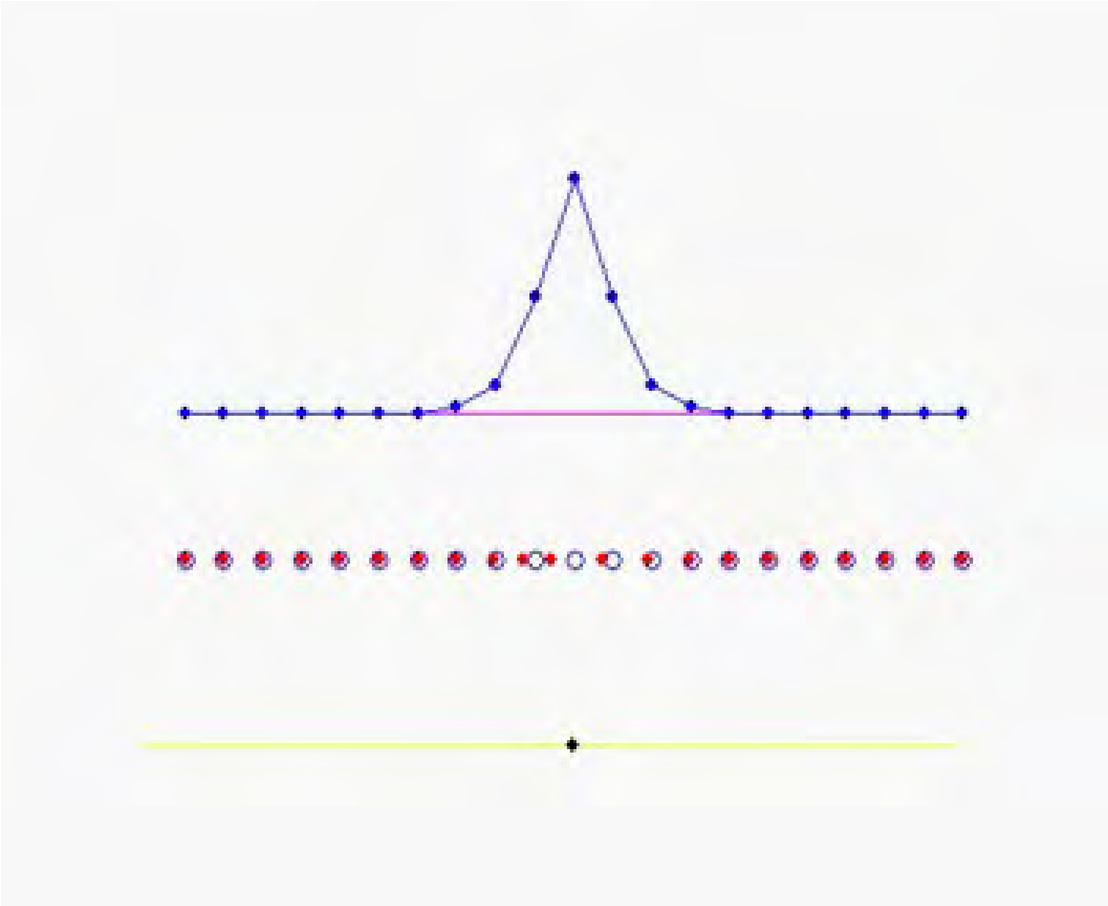








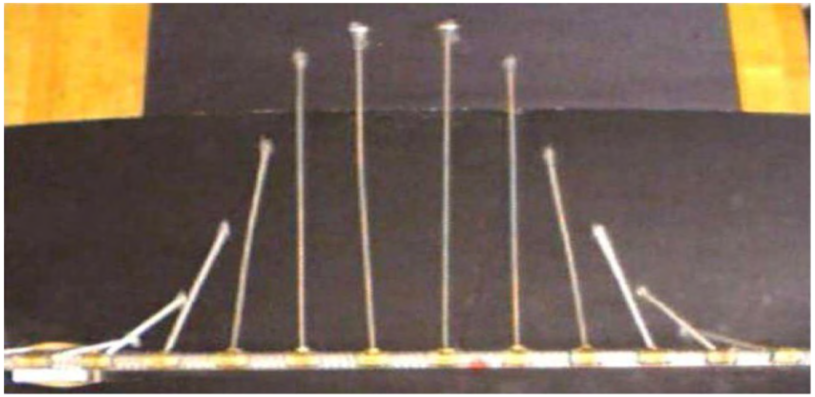
..and moving breathers



Are discrete breathers real?

Experimental results

Antiferromagnets, charge-transfer solids, crystals, photonic crystals, superconducting Josephson junctions, macroscopic mechanical systems, micromechanical cantilever arrays, granular crystals, biopolymers, biomolecules?...



Direct control and manipulation.
Experimental techniques!!

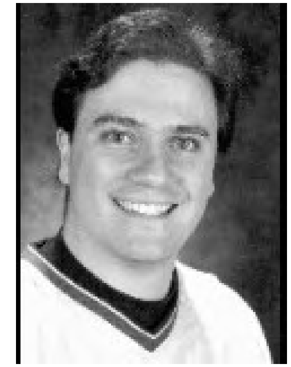
Experiments and theory, how to compare?

Our “rat lab”: a macroscopic family of electrical lattices

Electrical lattices

A. J. Sievers. Laboratory of Atomic and Solid Physics, Cornell University Ithaca, NY, USA

Panayotis G. Kevrekidis. Department of Mathema tics and Statistics , University of Amherst, MA, USA



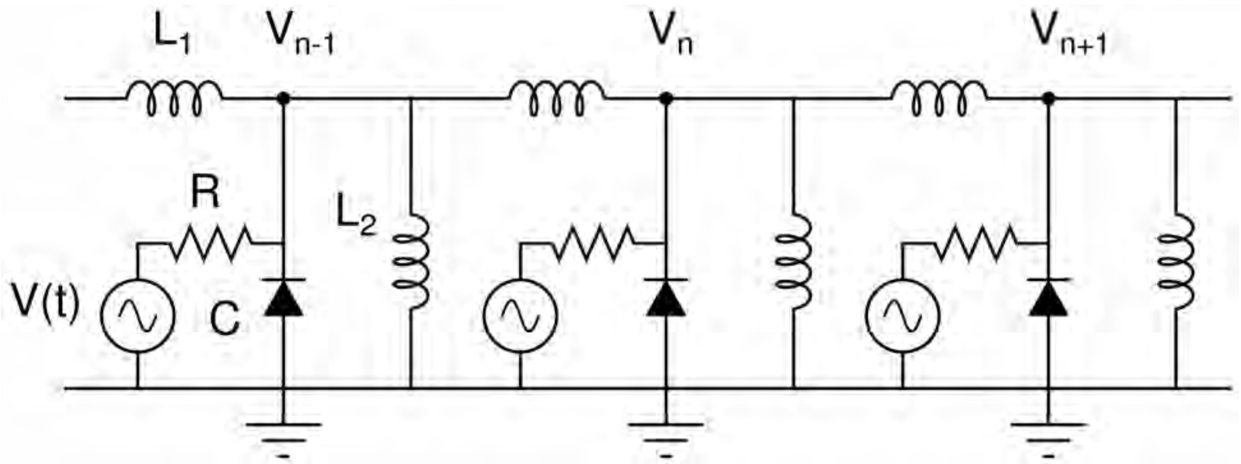
Ricardo Carretero. Department of Mathematics and Statistics . San Diego State University, San Diego, CA,USA

D.H. Barnak , P. Candiani, R.B. Thakur , R. Stearrett and J. Stormes. Physics and Astronomy Department. Dickinson College, PA, USA

Lars Q. English. Physics and Astronomy Department . Dickinson College, PA, USA

Jesús Cuevas and Faustino Palmero. Group of Nonlinear Physics, University of Seville, Spain

Electrical lattices, why?



Using a device made of discrete components and some nonlinear elements we can combine dispersion and nonlinearity. You can get discrete breathers!

You only need basic physics to understand it!

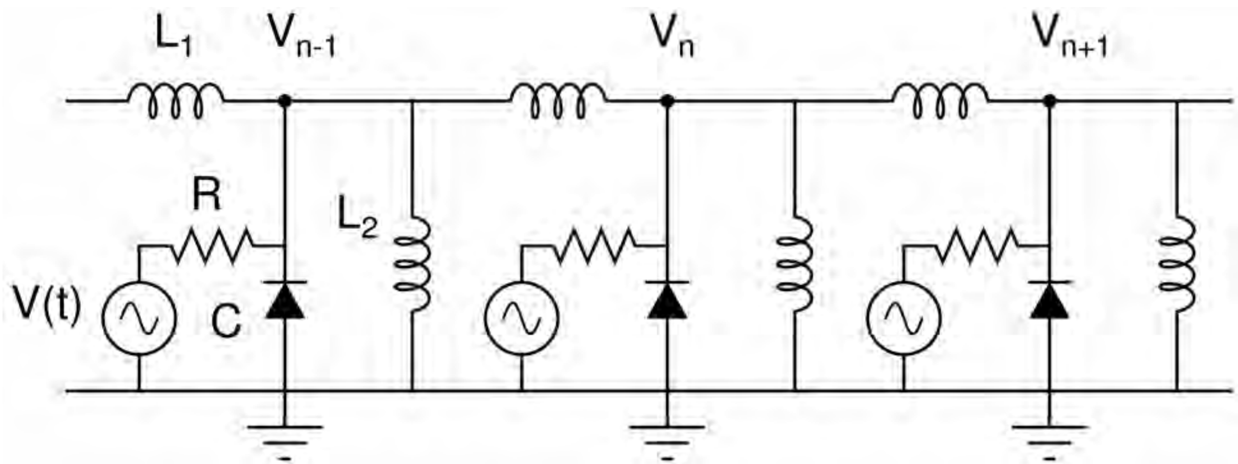
We havedeveloped a method of achieving spatial control of discrete breathers

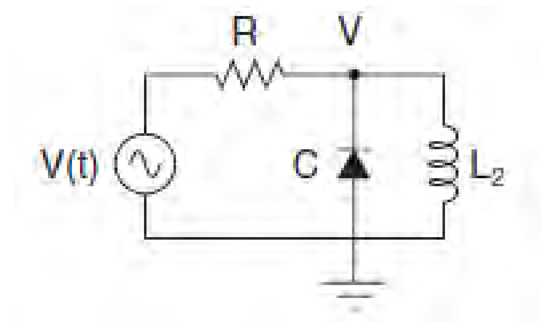
You can easily build them experimentally (and also, it is cheap!).

We can study experimentally discrete breathers with a spatial scale which allows for the detailed characterization of its profiles, with is usually only possible in numerical simulations

Ideal "lab rat" to styudy, theoretical and experimentally, discrete breathers in one, two, (and three?) dimensional systems

Electrical line





32 nodes. Ring structure (p.b.c.)

Nonlinear element: varactor diode NTE 618 (C(0)≈800 pF)

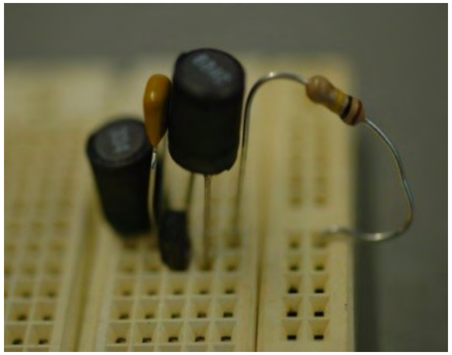
Inductors L1=680 μH and L2=330 μH

No dc biass

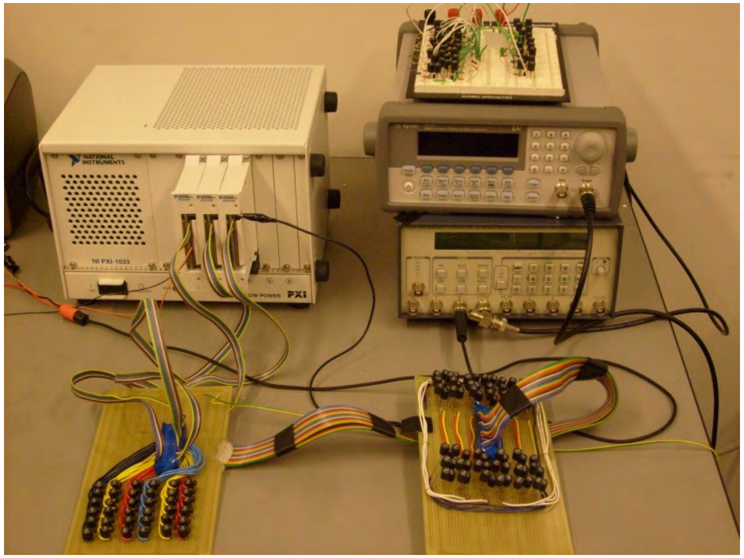
R=10 kΩ. V(t)=Vdcos(ωt), where Vd varies from 1 V to 5V and frequencies from f=200 kHz to f=600 kHz

Observable: tension Vn

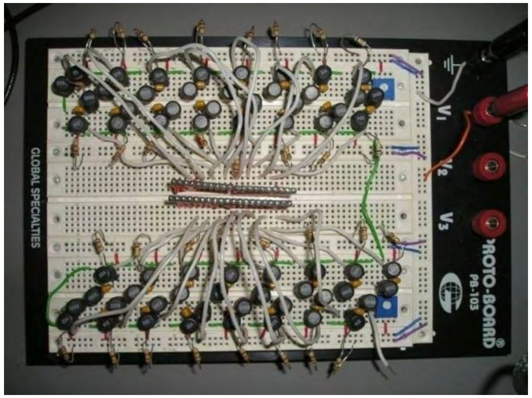
“Real” electrical line



Unit cell



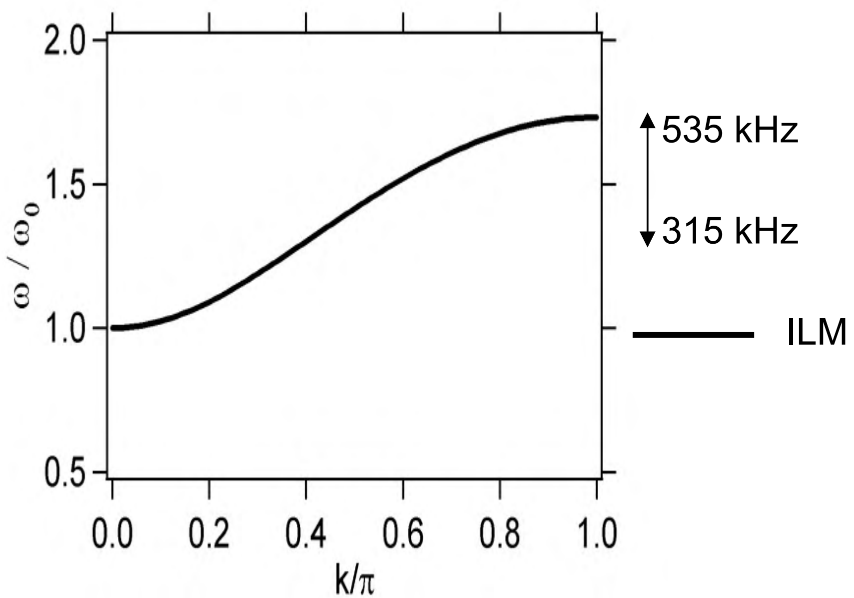
Fast 32-channel digitizer. Voltage monitores at 0.4 μs



Generating discrete breathers

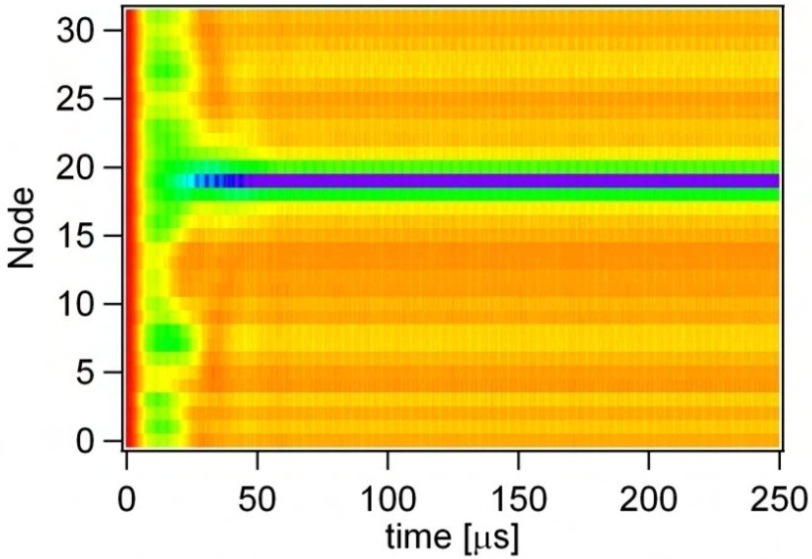
Low amplitude plane-wave dispersion curve

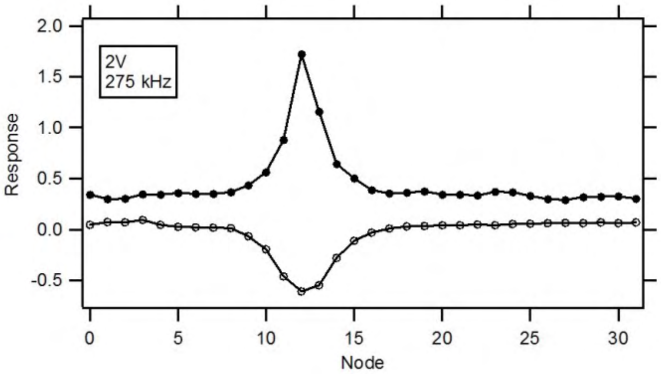
$$\omega^2 = \frac{1}{L_2 C} + \frac{4}{L_1 C} \sin^2 k/2$$

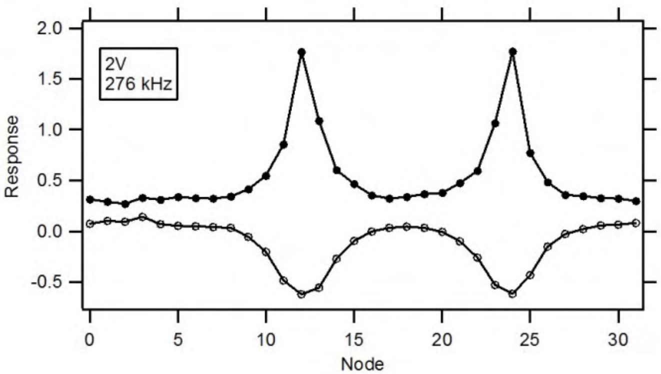


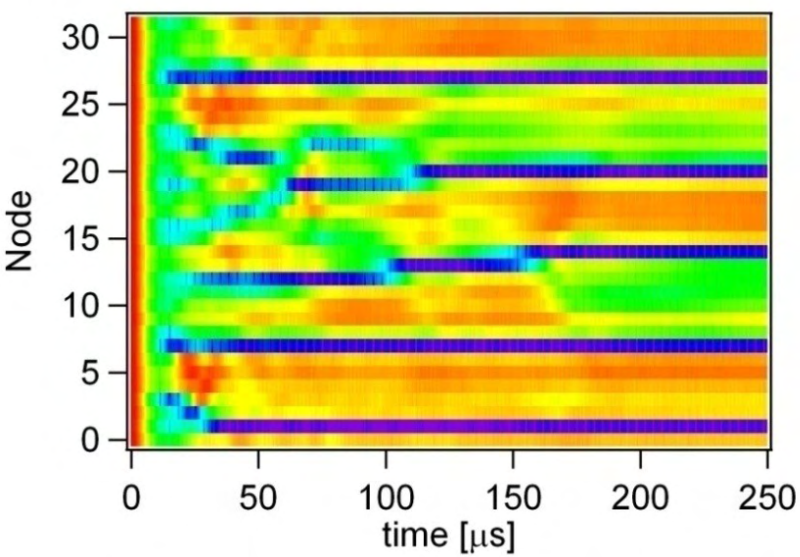
Modulational instability: for a frequency gapped linear spectrum, some of the linear modes become unstable at large amplitude. We consider the uniform mode (k=0)

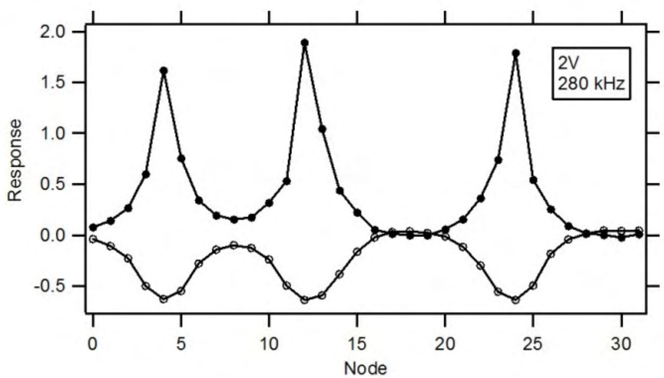
Experimental results



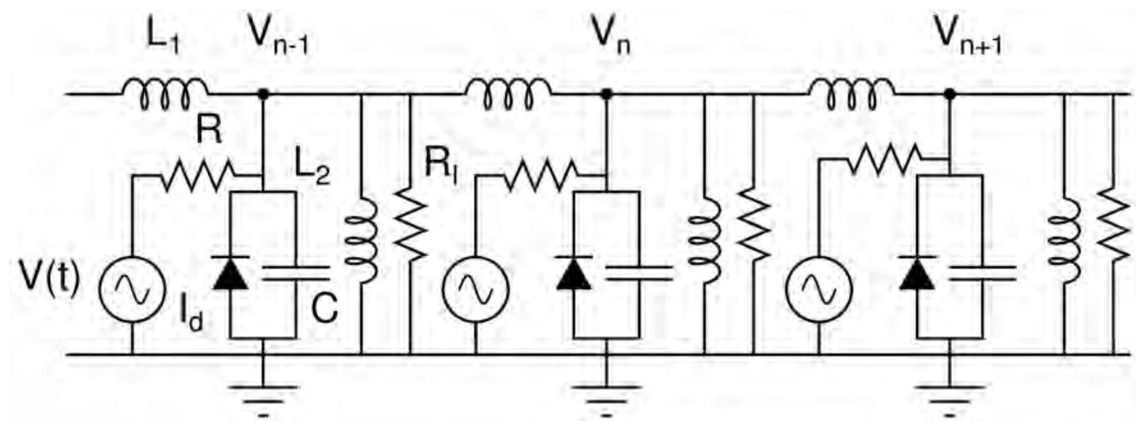


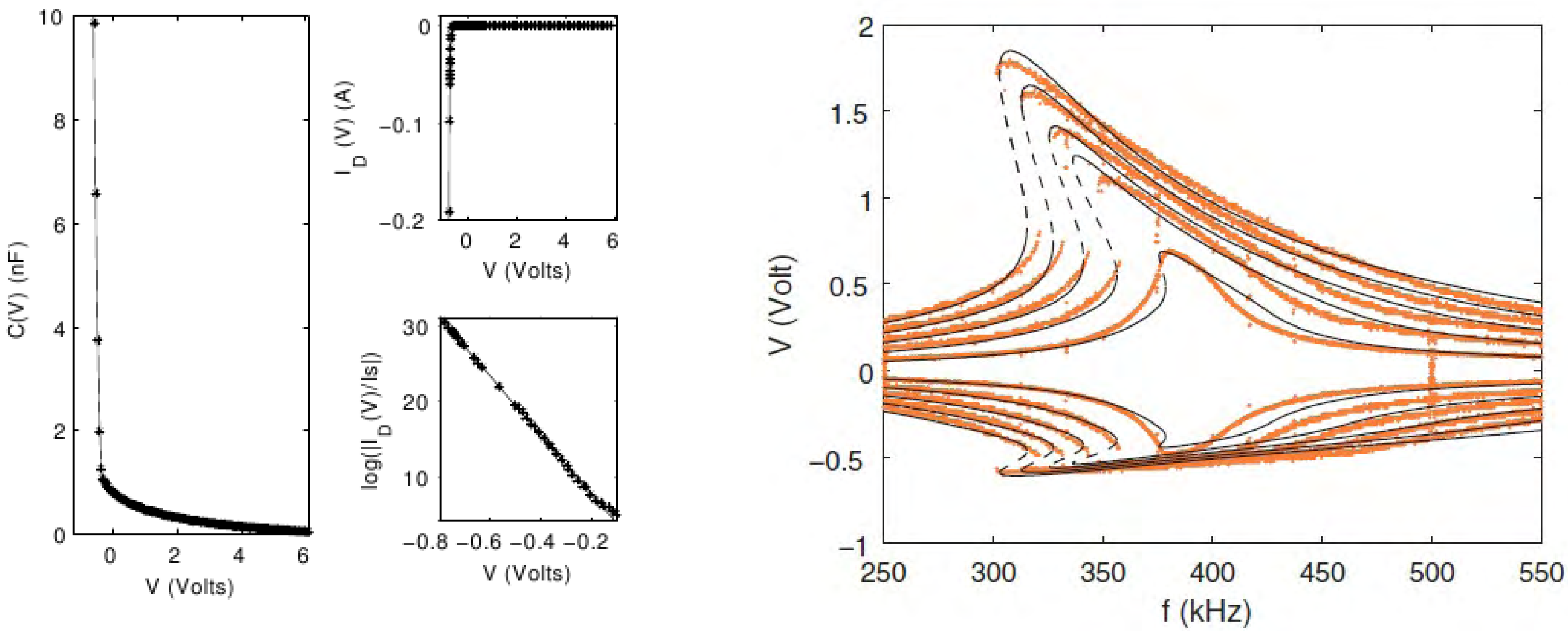






Theoretical model





uations and numerical methods

$$c(v_n)\frac{dv_n}{d\tau} = y_n - i_D(v_n) + \frac{\cos(\Omega\tau)}{RC_{0\omega_0}} - \left(\frac{1}{R_l} + \frac{1}{R}\right)\frac{v_n}{\omega_0 C_0}$$

$$\frac{dy_n}{d\tau} = \frac{L_2}{L_1}(v_{n+1} + v_{n-1} - 2v_n) - v_n$$

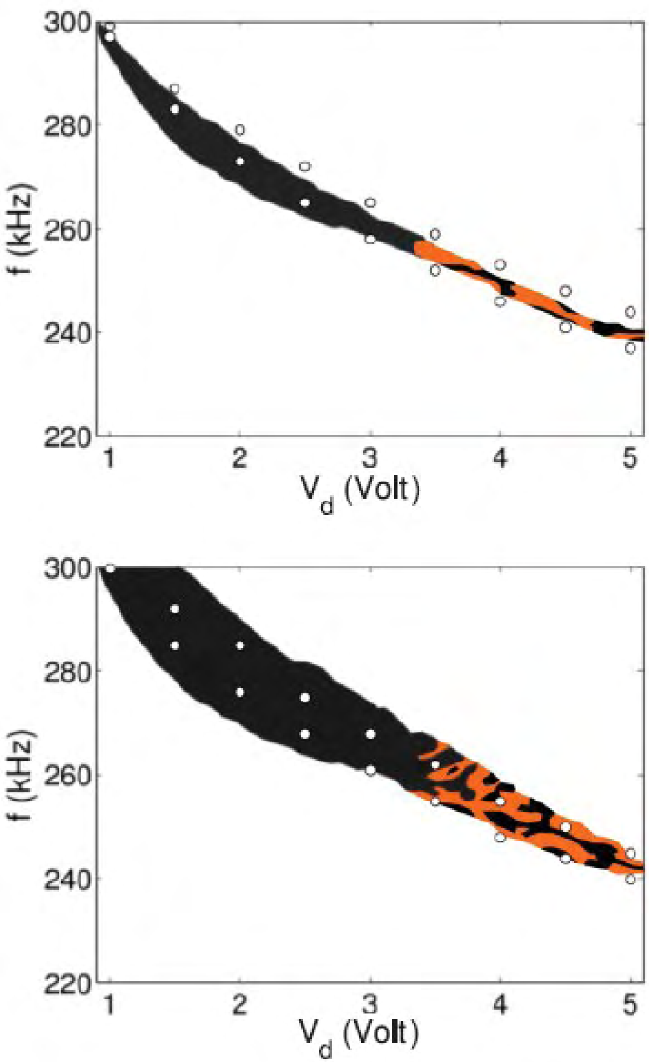
Discrete breathers: fixed points of the map

$$\begin{bmatrix} v_n(0) \\ \frac{dv_n}{d\tau}(0) \\ y_n(0) \\ \frac{dy_n}{d\tau}(0) \end{bmatrix} \rightarrow \begin{bmatrix} v_n(T) \\ \frac{dv_n}{d\tau}(T) \\ y_n(T) \\ \frac{dy_n}{d\tau}(T) \end{bmatrix}$$

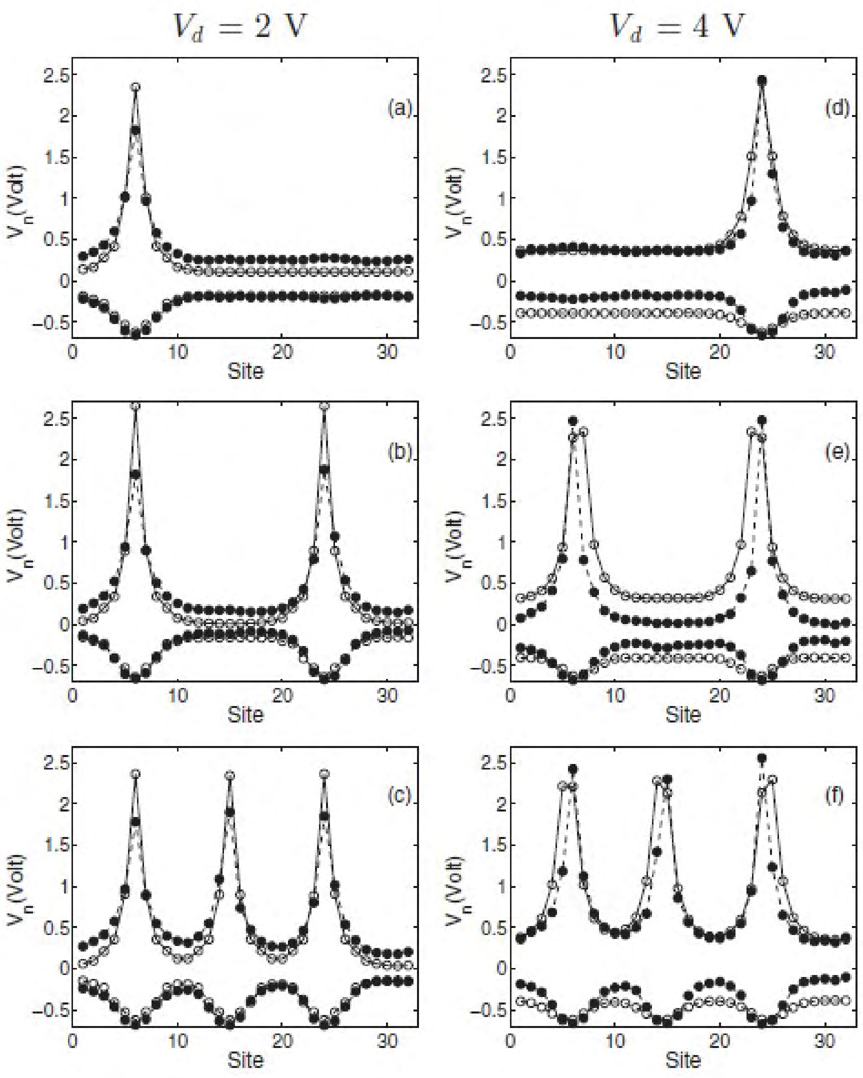
Stability given by the spectrum of the Floquet operator (4*N* X 4*N* matrix)

$$\begin{bmatrix} \xi_n(T) \\ \frac{d\xi_n}{d\tau}(T) \\ \eta_n(T) \\ \frac{d\eta_n}{d\tau}(T) \end{bmatrix} = \mathcal{M} \begin{bmatrix} \xi_n(0) \\ \frac{d\xi_n}{d\tau}(0) \\ \eta_n(0) \\ \frac{d\eta_n}{d\tau}(0) \end{bmatrix}$$

Results

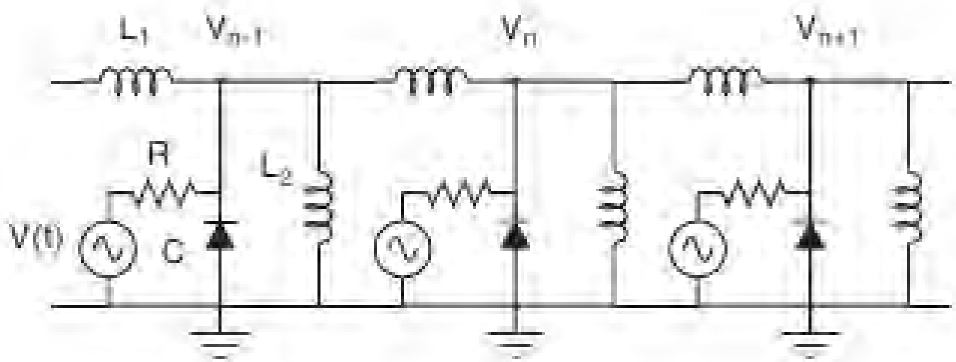


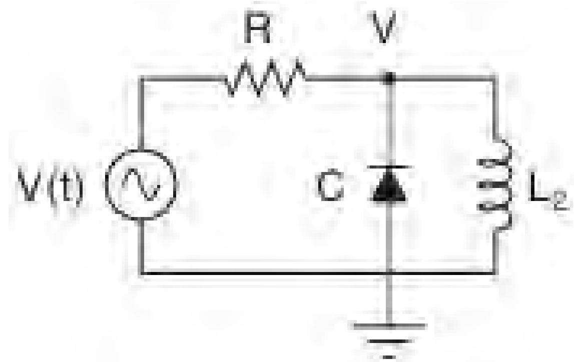
300
280
260
240
220
f (kHz)
1
2
3
4
5
V_d (Volt)

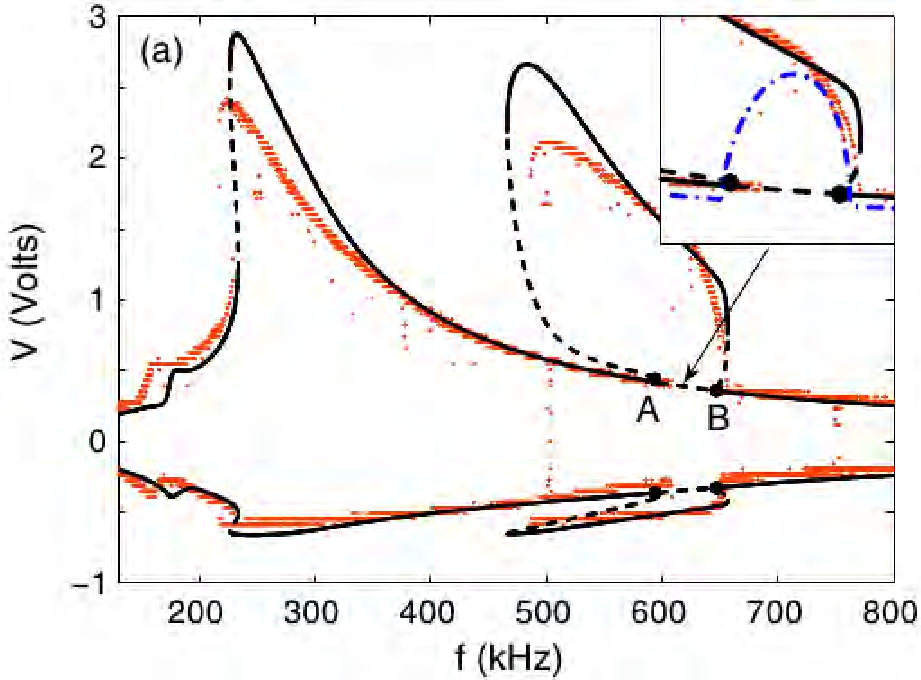


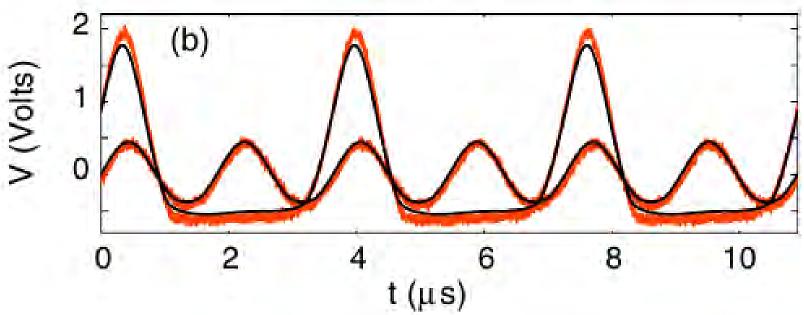
$V_d = 2$ V
$V_d = 4$ V
(a)
(b)
(c)
(d)
(e)
(f)
V_n (Volt)
Site

"Subharmonic" breathers



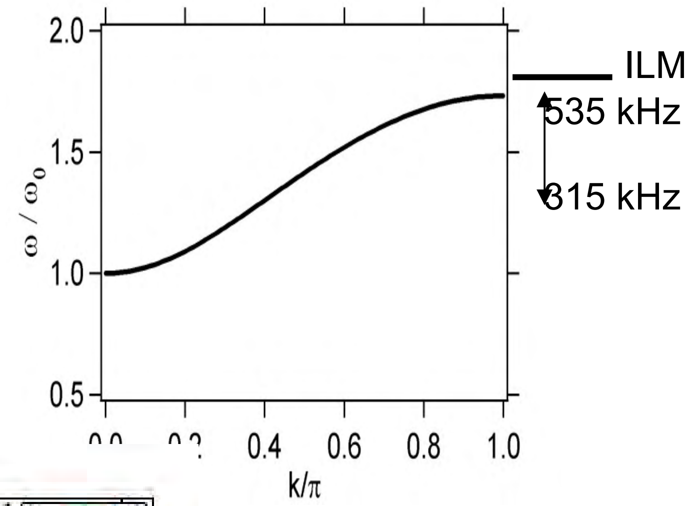


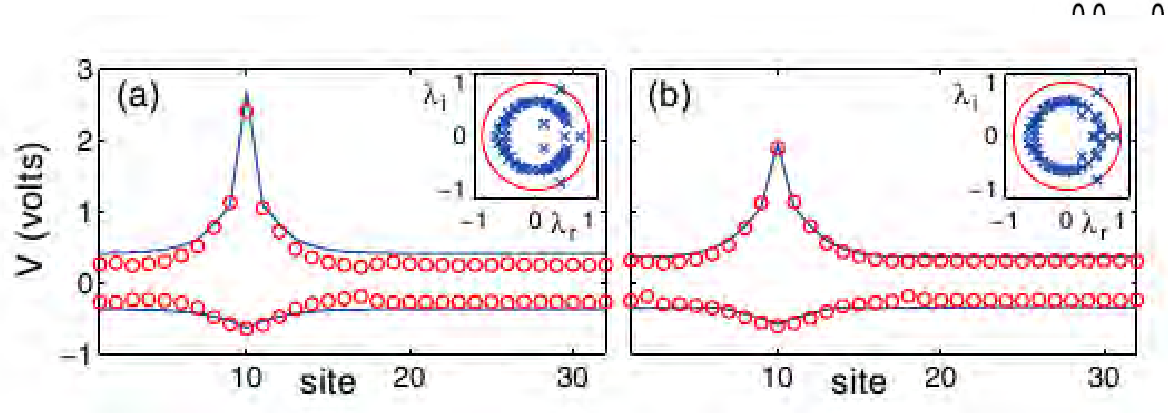


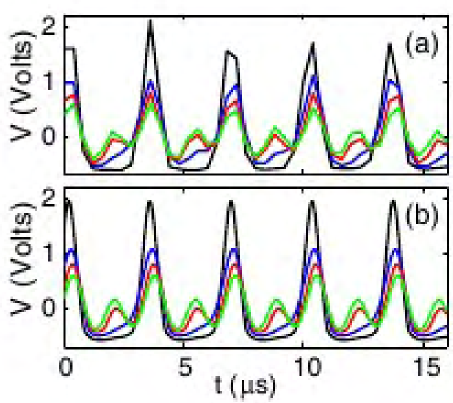


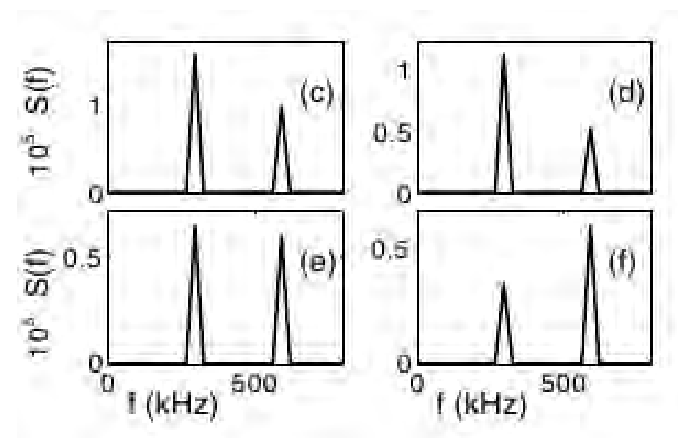
Results

Generation: Modulational instability

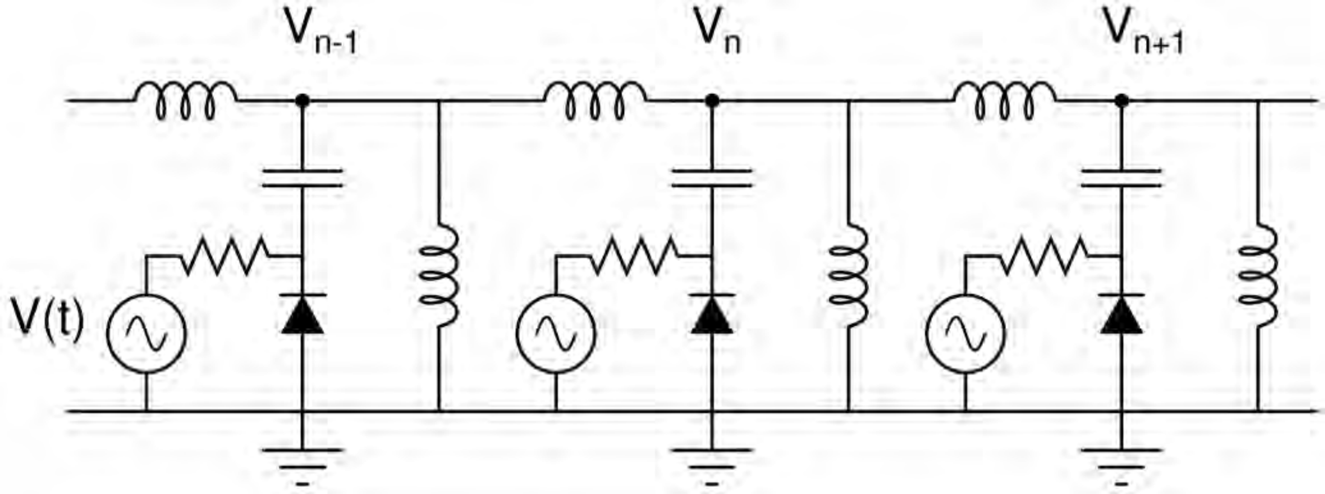








ectrical lattice with a “block” capacitor

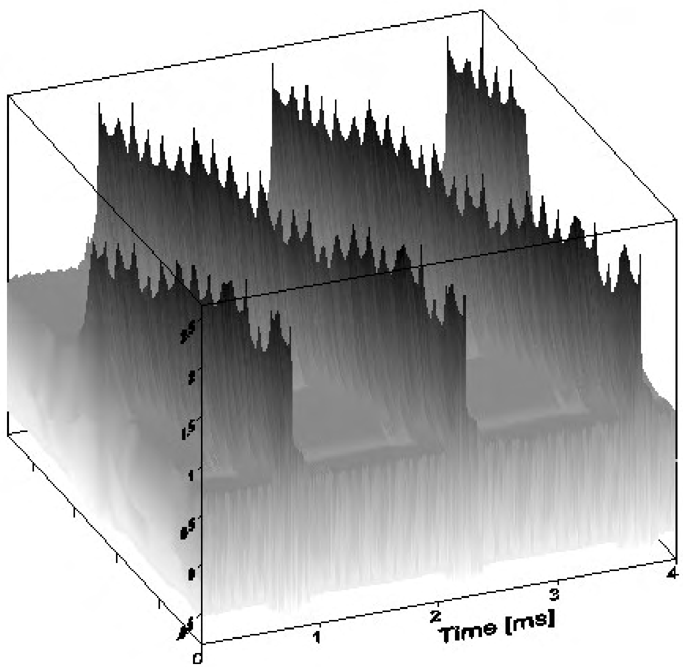


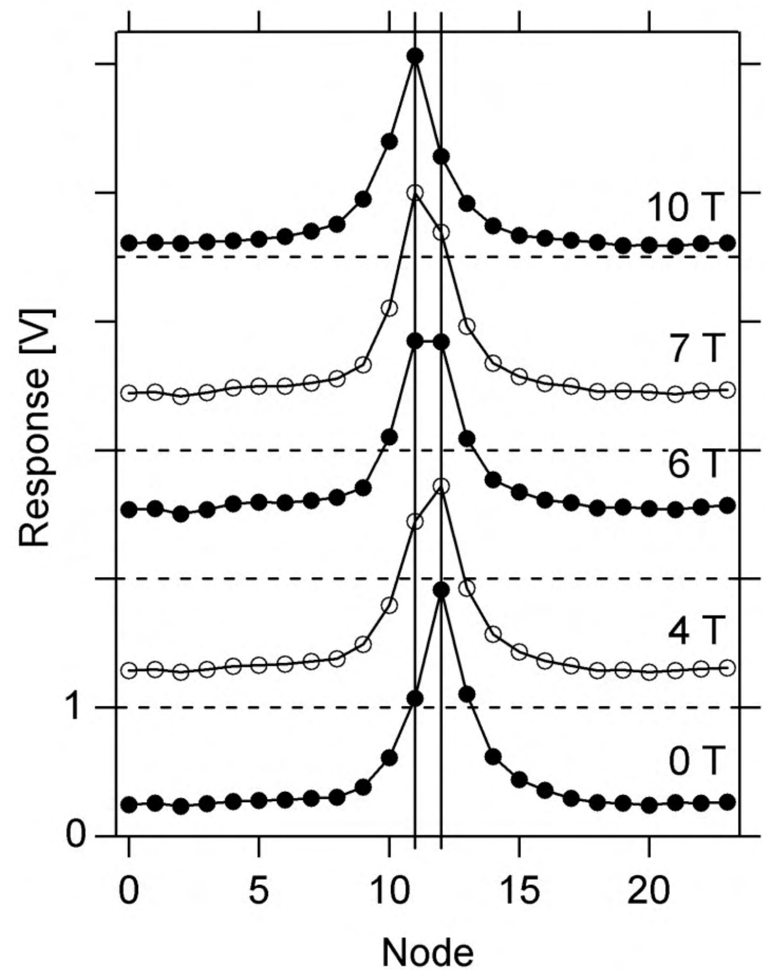
Blocking capacitor of 1 μF. Relative capacitance ratio of over a 1000. Impedance negligible. Does it have any effect?

No?...

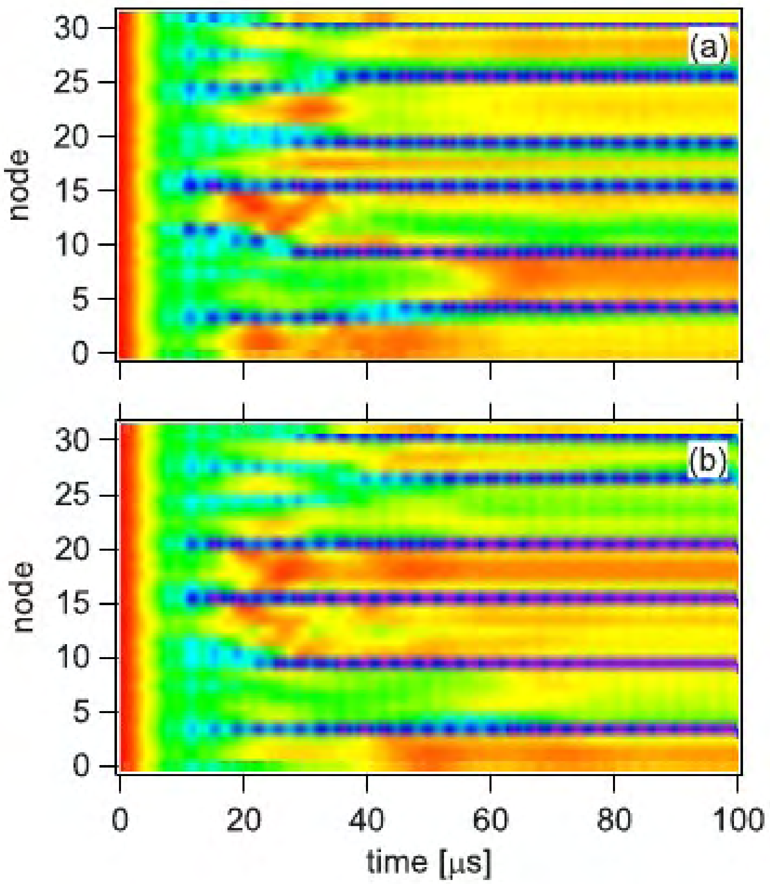
The answer is YES!!!

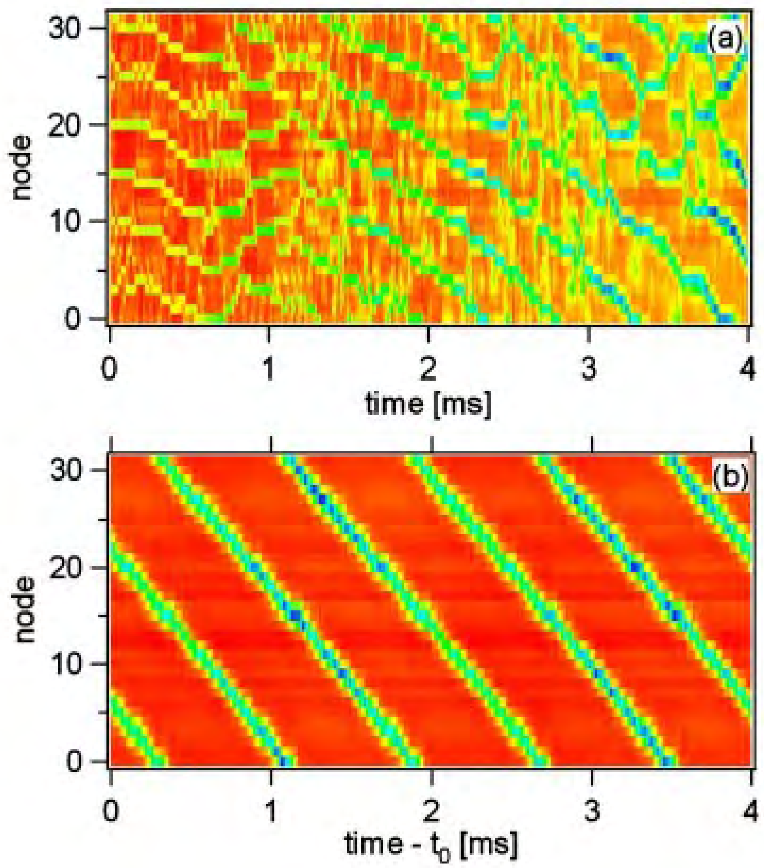
Moving breathers



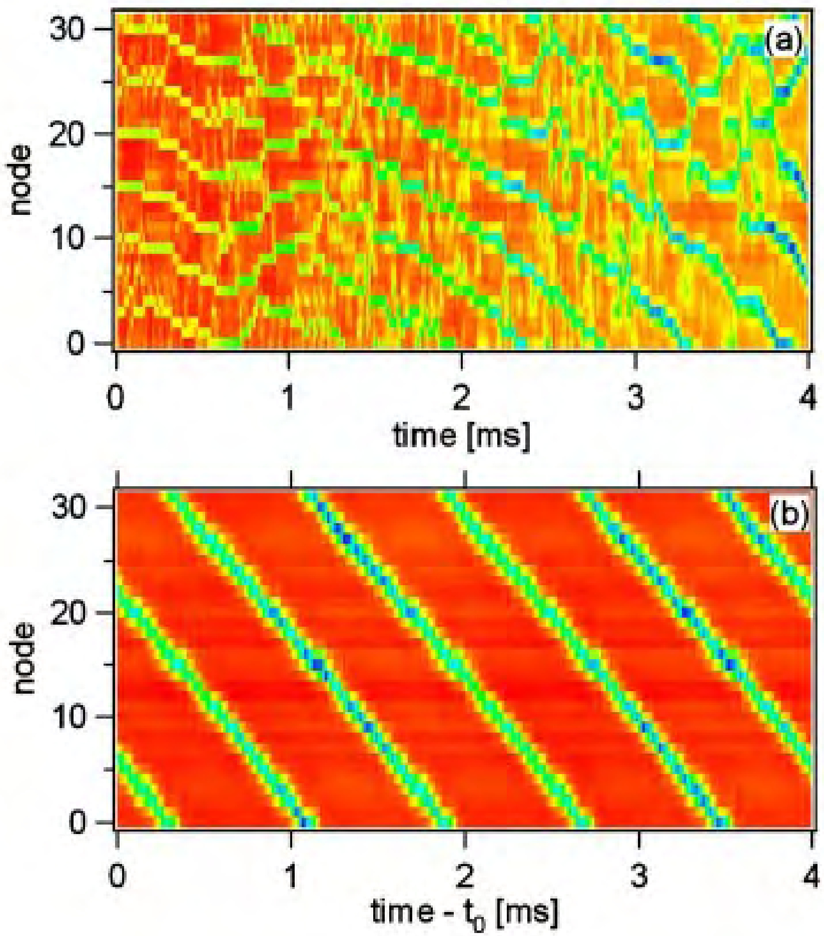


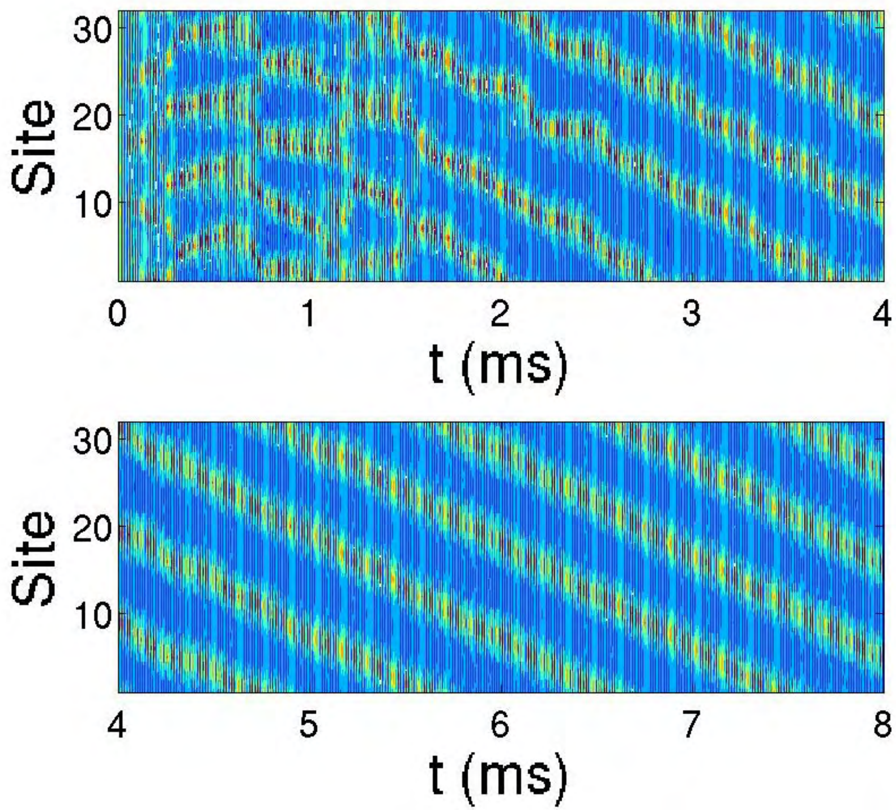
Transient state





xperimental and numerical results



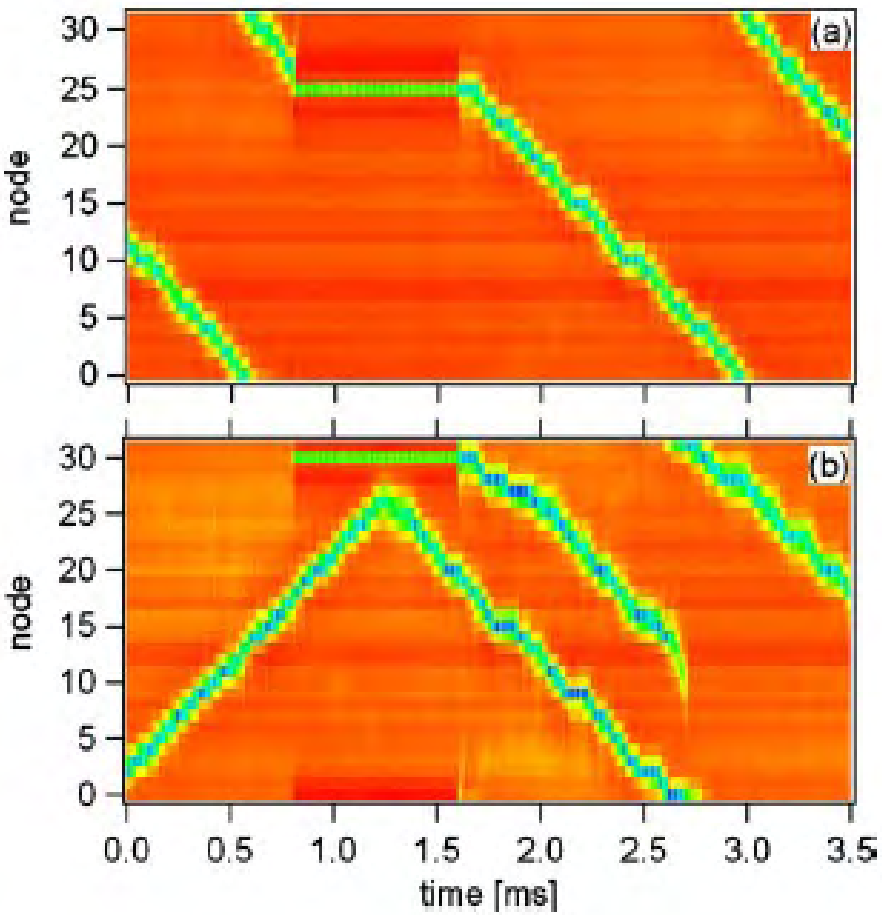


No bad
agreement

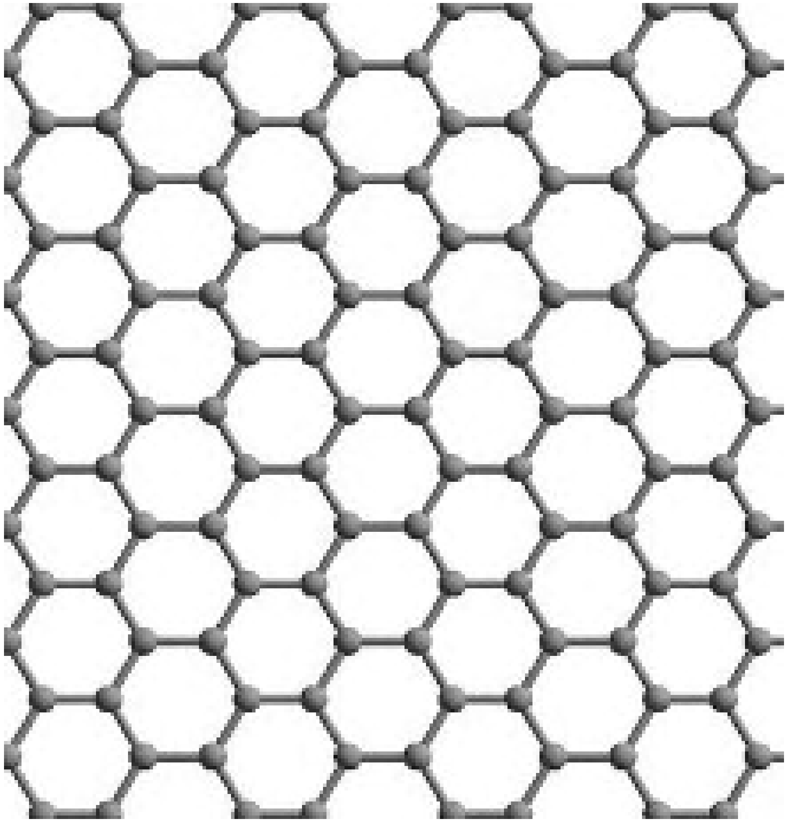
Control and manipulation

Control: Creation of an impurity at a site chosen as the ILM center. We can produce a ILM at a lattice site of our choice

Capture and release

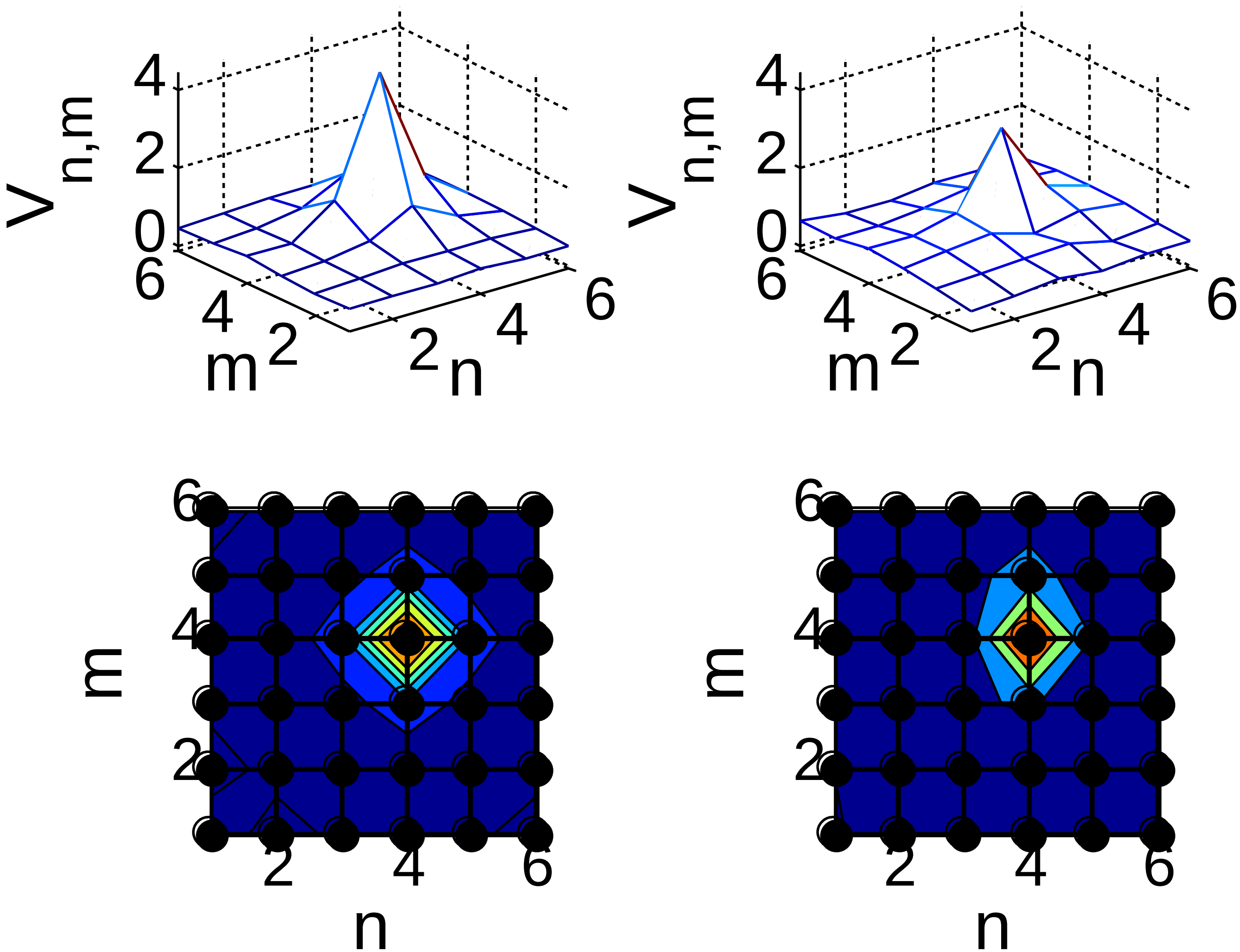


at we have been doing this summer. Last results

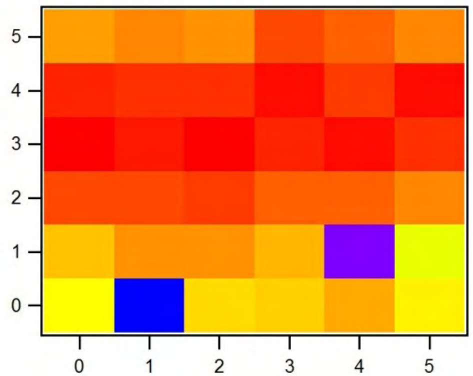


Experimental setup: 6 x 6 lattice!

Square lattice

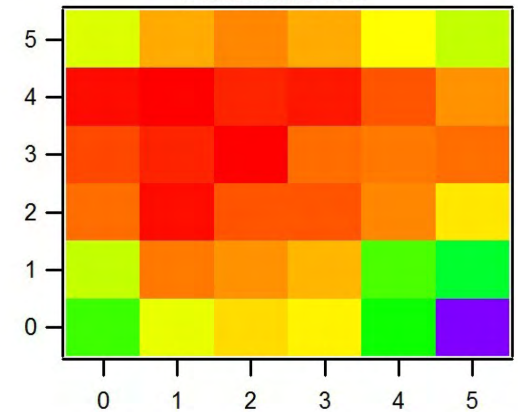


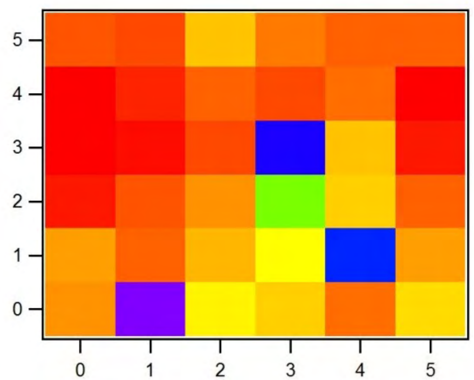
Square lattice



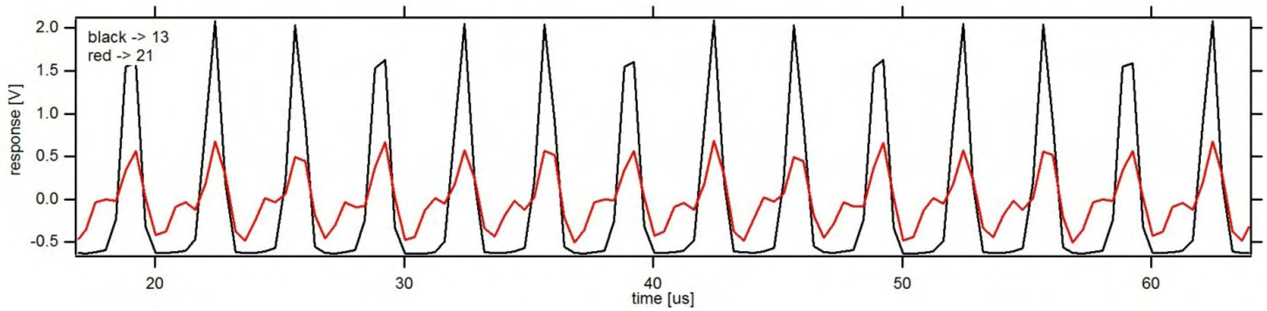
f=300 kHz, A=3 V

f = 600 kHz, A=7.5V
Subharmonic ILM

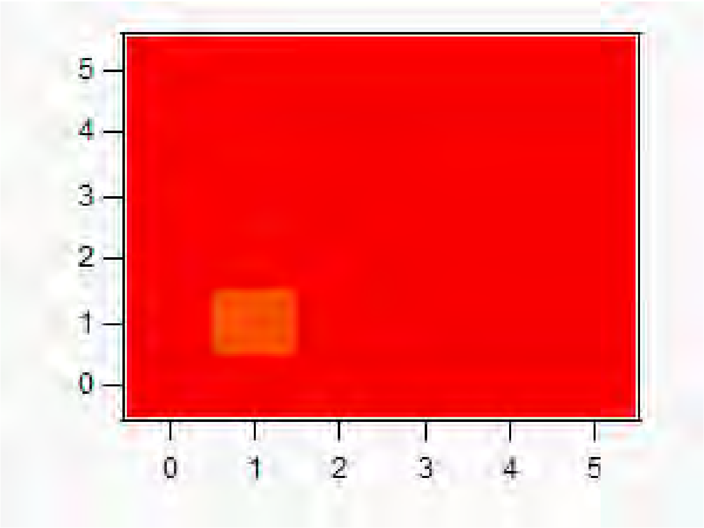


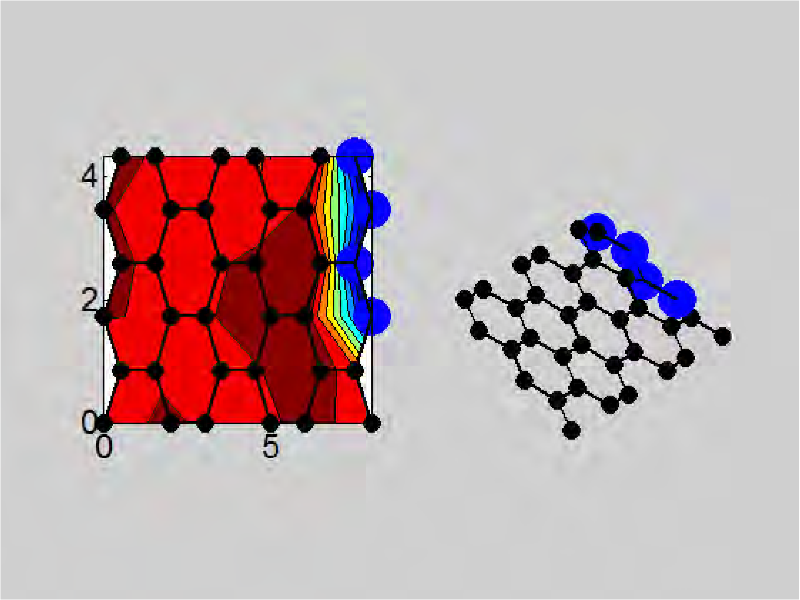


f=300 kHz, A=4 V



Moving breathers





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Thanks!