Discrete solitons in a one–dimensional nonlinear Schrödinger equation with a single inhomogeneity.

Faustino Palmero Acebedo. Nonlinear Physics Group.

Department of Applied Physics I. University of Seville, Spain

palmero@us.es

http://www.pdipas.us.es/p/palmero/

In collaboration with the group of Professor R. Carretero. Department of Mathematics and Statistics, SDSU, CA (USA).

Summary

- Introduction.
- Stationary solutions.
    - Linear modes.
    - Nonlinear modes.
        - Existence.
        - Stability.
        - Bifurcations.
- Nucleation problem.
- Interaction of moving localized modes with an impurity.
- Work in progress and perspectives.

Introduction

The model:

- Discrete one–dimensional lattice with a single inhomogeneity. DNLS:

$$i\dot{\psi}_n + \gamma|\psi_n|^2\psi_n + C(\psi_{n+1} + \psi_{n-1}) + \alpha_n\psi_n = 0,$$

- Two dynamical invariants

$$H = -\sum_n \frac{\gamma}{2}|\psi_n|^4 + C(\psi_n^*\psi_{n+1} + \psi_n^*\psi_{n-1}) + \alpha_n\psi_n^*\psi_n,$$

with canonical variables $q_n = \psi_n$ and $p_n = i\psi_n^*$, and the norm

$$P = \sum_n |\psi|^2.$$

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Stationary states

- Stationary solutions: $\psi_n = e^{i\omega t}\phi_n$
- Equation:

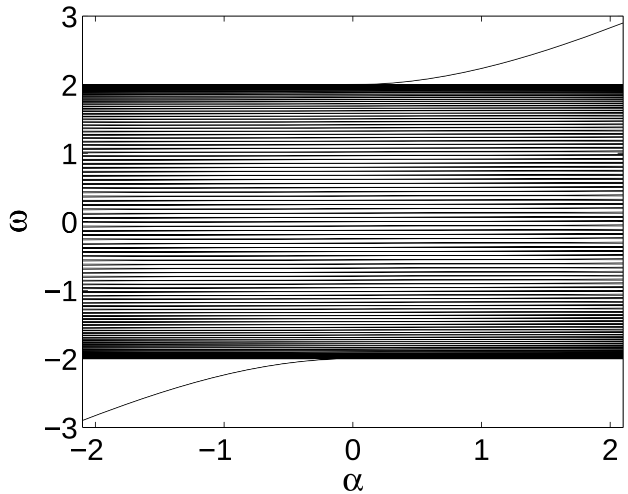
$$-\omega\phi_n + C(\phi_{n+1} + \phi_{n-1}) + \phi_n^3 + \alpha_n\phi_n = 0.$$

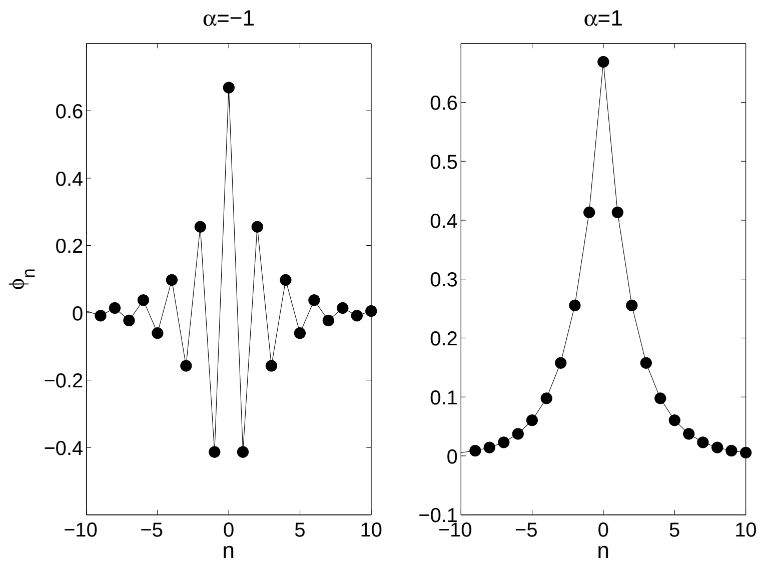
- Linear modes:

$$\begin{bmatrix} \alpha & C & 0 & . & . & C \\ C & 0 & C & 0 & . & 0 \\ 0 & C & 0 & C & . & . \\ . & . & . & . & . & . \\ . & . & . & C & 0 & C \\ C & . & . & . & C & 0 \end{bmatrix} \begin{bmatrix} \phi_0 \\ \phi_1 \\ . \\ . \\ \phi_{N-2} \\ \phi_{N-1} \end{bmatrix} = \omega \begin{bmatrix} \phi_0 \\ \phi_1 \\ . \\ . \\ \phi_{N-2} \\ \phi_{N-1} \end{bmatrix},$$

Stationary states. Linear modes

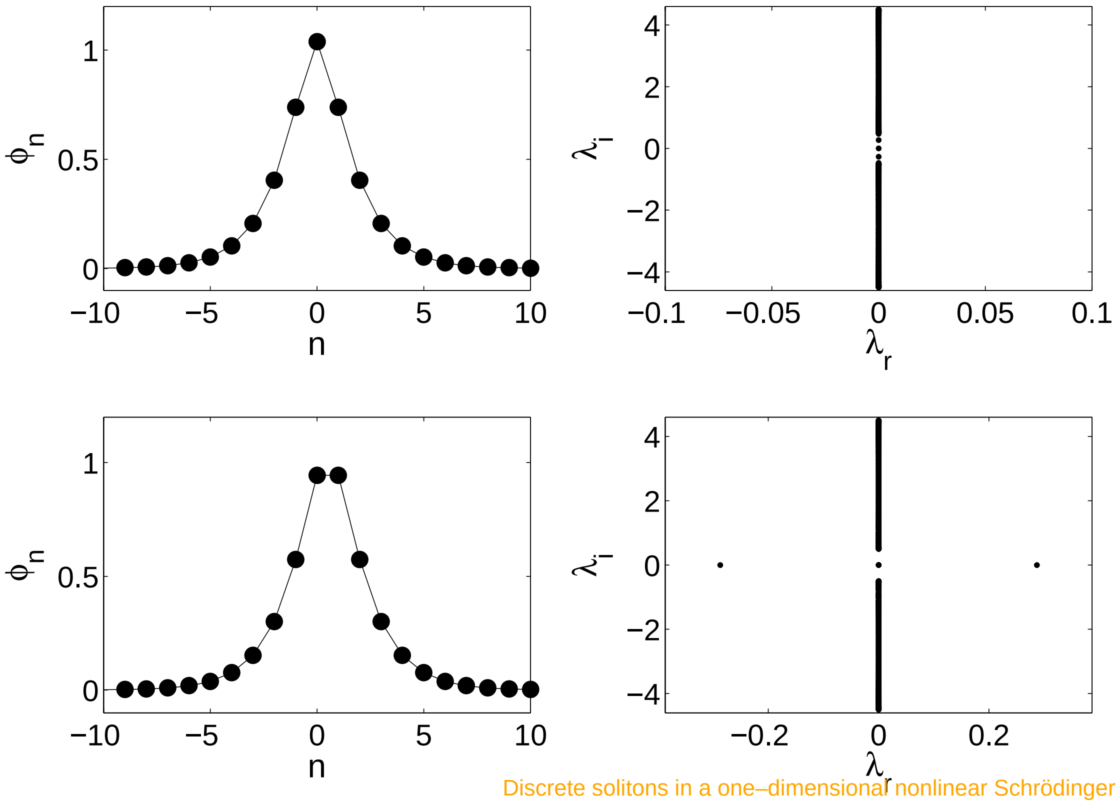
- Analytical expressions



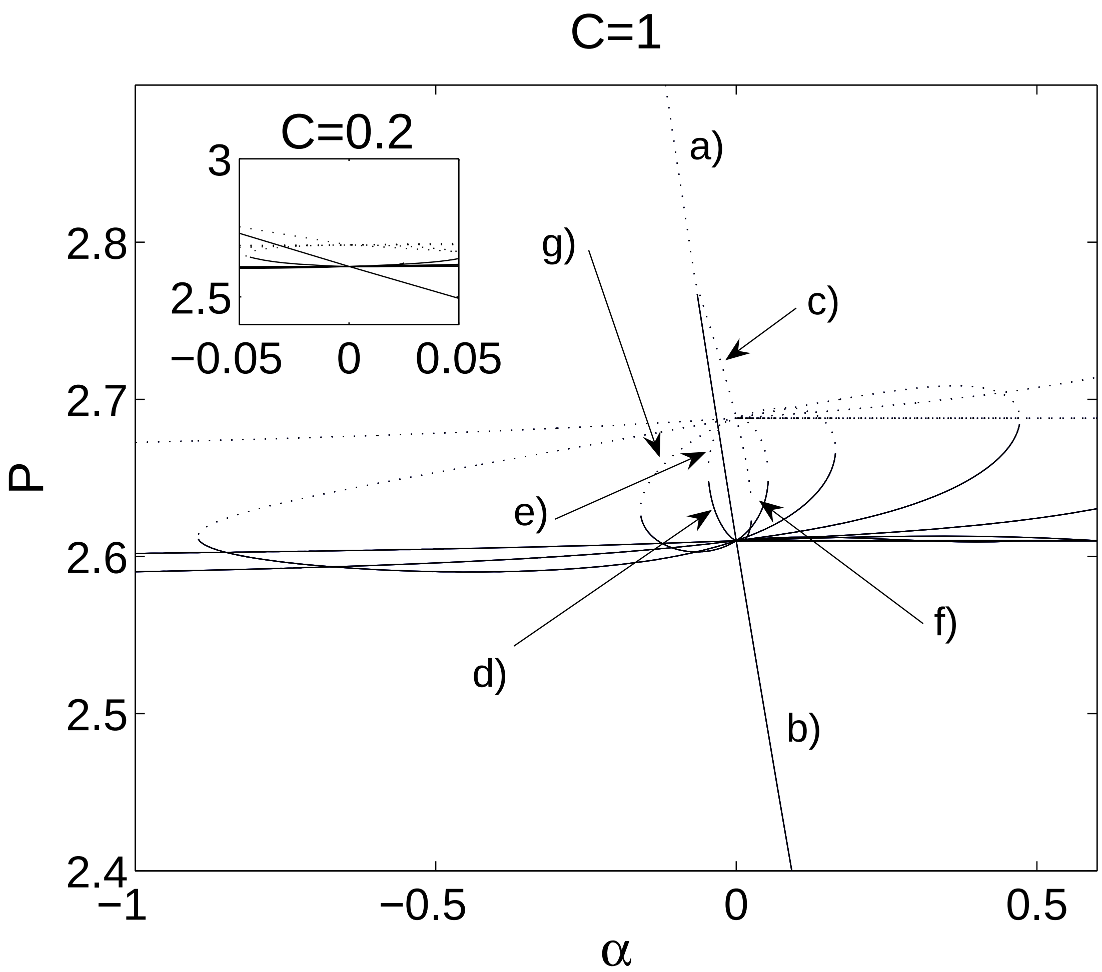


Stationary states. Nonlinear modes

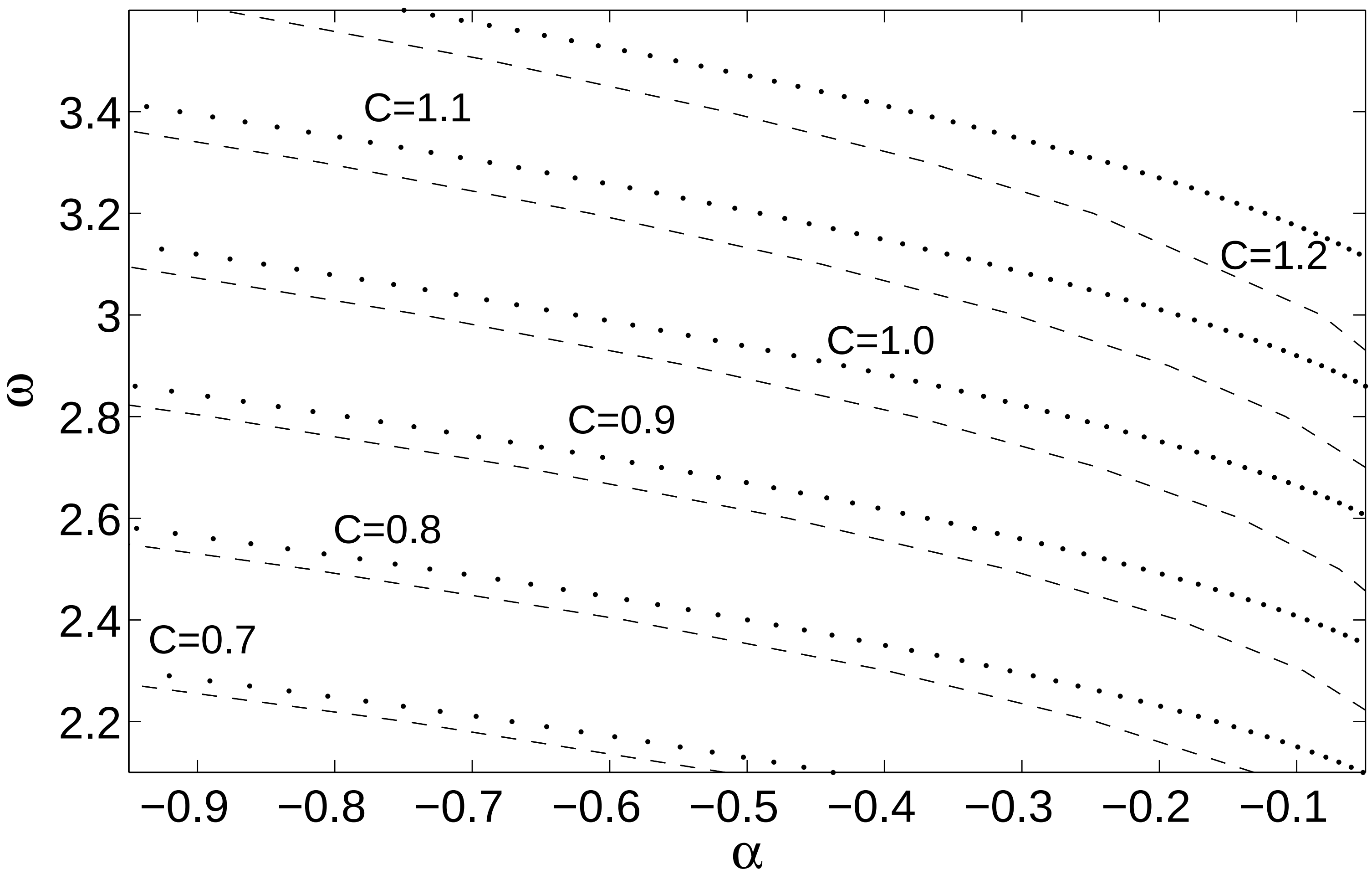
- Solutions equation.
- Stability: $\psi_n = [\phi_{sol} + \epsilon(a_n \exp(\lambda t) + b_n \exp(\lambda^* t)] \exp(i\omega t)$.



Stationary States. Bifurcations



Bifurcations II



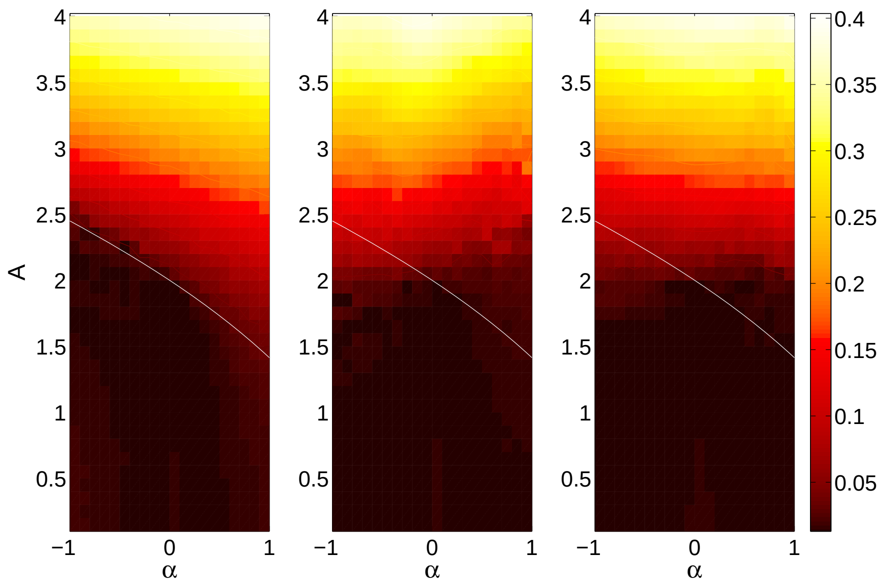
Nucleation

- Energy threshold to have localized modes in this system. Hypothesis:

$$(2C - \alpha)A^2 - \gamma/2A^4 < 0.$$

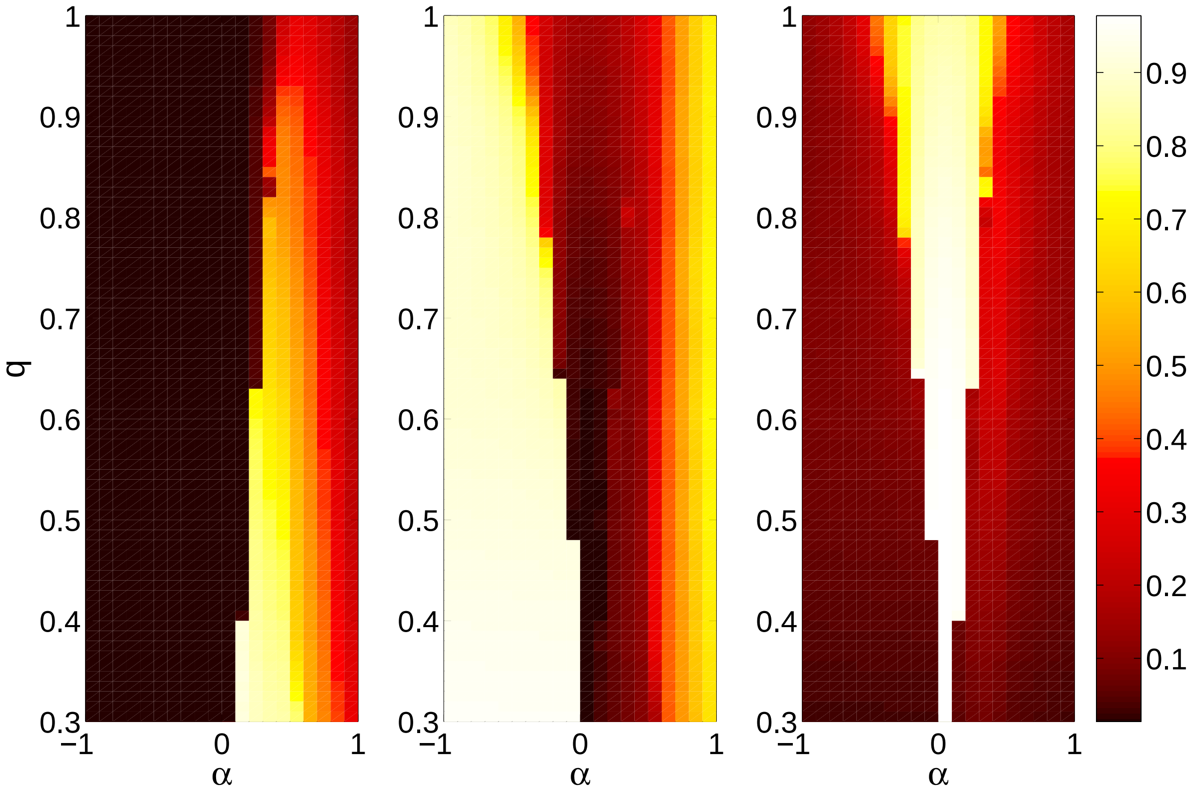
- Localization parameter

$$L(E) = \frac{\sum_n |\varphi_n|^2}{(\sum |\varphi_n|)^2}.$$

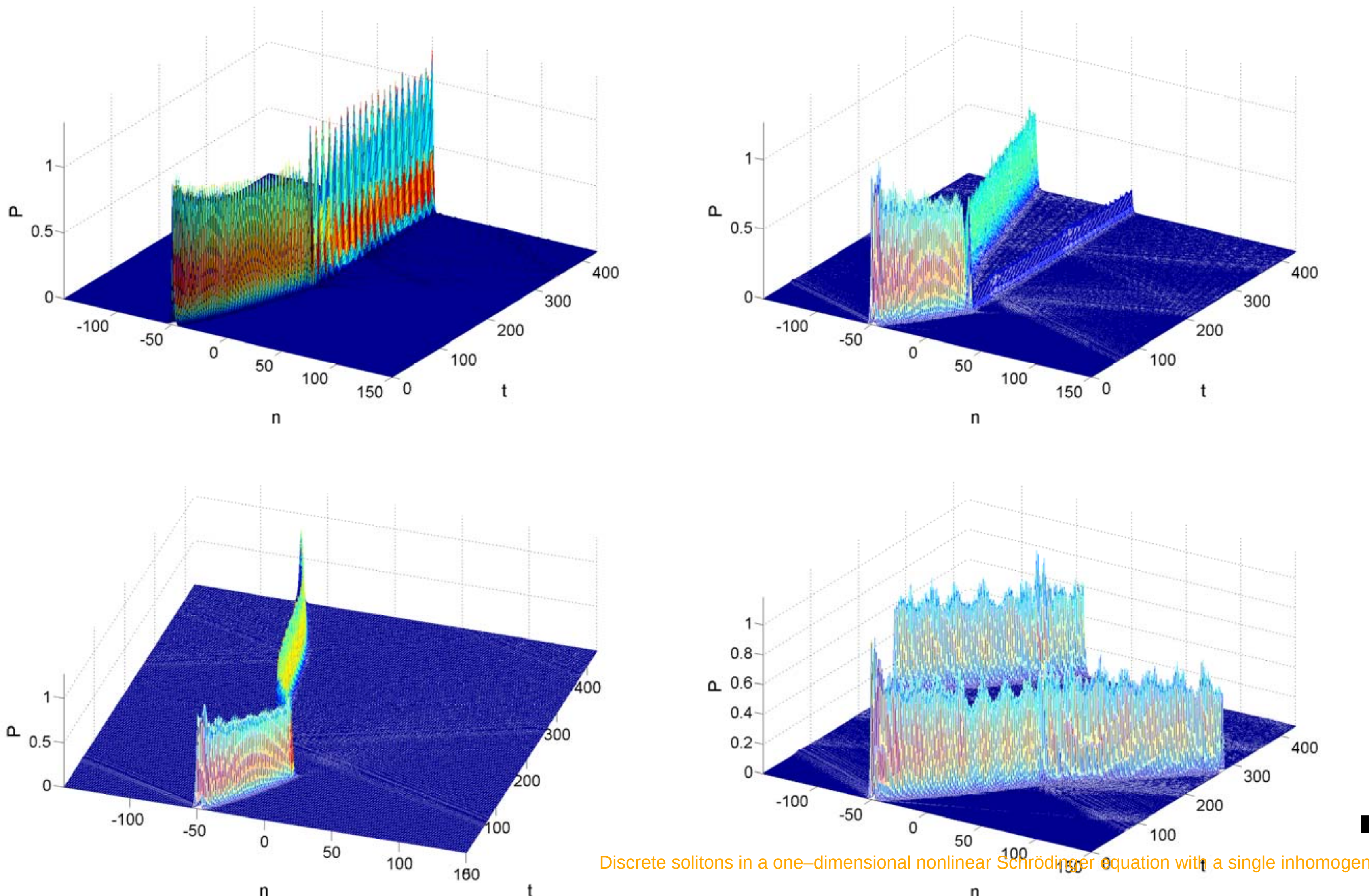


Moving breathers

- Moving stationary states: $\varphi_n(t=0) = \phi_n exp(iqn)$.



Moving breathers II



Future work

- Quantum equivalence
- Two–dimensional system
- More information:
  - Contact with Prof. Carretero
  - J.C. Eilbeck and M. Johansson. ”The discrete Nonlinear Schrödinger equation– 20 years on". Proceedings of the Third Conference Localization and Energy Transfer in Nonlinear Systems”, June 17-21, San Lorenzo de El Escorial Madrid, eds L. Vázquez, R. S. MacKay, M. P. Zorzano, World Scientific, Singapore, 44–67, 2003. (draft) http://www.ma.hw.ac.uk/ chris/em02.pdf