Analytical approximations to discrete soliton profiles in DNLS models

Jesús Cuevas–Maraver

Nonlinear Physics Group. University of Seville (Spain)
http://www.grupo.us.es/gfnl

NEEDS 2007. June 22, 2007

- This work has been done in collaboration with:
  - Panayotis G. Kevrekidis (University of Massachussets)
  - Boris A. Malomed (Tel-Aviv University)
  - Dimitri J. Frantzeskakis (University of Athens)
  - Guillaume James (INSA de Toulouse)
  - Bernardo Sánchez–Rey (Universidad de Sevilla)
  - Faustino Palmero (Universidad de Sevilla)

  to whom I am very acknowledged.

Outline

1. Introduction

Outline

Outline

Outline

The Discrete Nonlinear Schrödinger (DNLS) equation

and its relation with the Nonlinear Schrödinger equation

- A very ubiquitous equation in nonlinear physics is the one-dimensional Nonlinear Schrödinger (NLS) Equation:

$$i\partial_t\psi + \partial_{xx}\psi + |\psi|^2\psi = 0$$

The Discrete Nonlinear Schrödinger (DNLS) equation

and its relation with the Nonlinear Schrödinger equation

- A very ubiquitous equation in nonlinear physics is the one-dimensional Nonlinear Schrödinger (NLS) Equation:

$$i\partial_t\psi + \partial_{xx}\psi + |\psi|^2\psi = 0$$

- The most famous discretizations of this equation are:

The Discrete Nonlinear Schrödinger (DNLS) equation

and its relation with the Nonlinear Schrödinger equation

- A very ubiquitous equation in nonlinear physics is the one-dimensional Nonlinear Schrödinger (NLS) Equation:

$$i\partial_t\psi + \partial_{xx}\psi + |\psi|^2\psi = 0$$

- The most famous discretizations of this equation are:
  - The Ablowitz-Ladiz equation:

$$i\dot{\psi}_n + (\psi_{n+1} + \psi_{n-1})(1 + |\psi_n|^2) = 0$$

The Discrete Nonlinear Schrödinger (DNLS) equation

and its relation with the Nonlinear Schrödinger equation

- A very ubiquitous equation in nonlinear physics is the one-dimensional Nonlinear Schrödinger (NLS) Equation:

$$i\partial_t\psi + \partial_{xx}\psi + |\psi|^2\psi = 0$$

- The most famous discretizations of this equation are:
  - The Ablowitz-Ladiz equation:

$$i\dot{\psi}_n + (\psi_{n+1} + \psi_{n-1})(1 + |\psi_n|^2) = 0$$

  - The DNLS equation:

$$i\dot{\psi}_n + (\psi_{n+1} + \psi_{n-1} - 2\psi_n) + |\psi_n|^2\psi_n = 0$$

The Discrete Nonlinear Schrödinger (DNLS) equation

Solutions of the Ablowitz-Ladik equation

- The Ablowitz-Ladik equation is integrable.

The Discrete Nonlinear Schrödinger (DNLS) equation

Solutions of the Ablowitz-Ladik equation

- The Ablowitz-Ladik equation is integrable.
- Discrete solitons solutions are given by:

$$\psi_n(t) = \frac{\sinh\beta}{\cosh[\beta(n - x_0(t))]} \exp[i\alpha(n - x_0(t)) + \theta(t)]$$

The Discrete Nonlinear Schrödinger (DNLS) equation

Solutions of the Ablowitz-Ladik equation

- The Ablowitz-Ladik equation is integrable.
- Discrete solitons solutions are given by:

$$\psi_n(t) = \frac{\sinh\beta}{\cosh[\beta(n-x_0(t))]}\exp[i\alpha(n-x_0(t))+\theta(t)]$$

- With:

$$v = \dot{x}_0(t) = \frac{2\sinh\beta\sin\alpha}{\beta}, \qquad \omega = \dot{\theta}(t) = 2\cosh\beta\cos\alpha + \alpha v$$

The Discrete Nonlinear Schrödinger (DNLS) equation

is not integrable

- On the contrary, the DNLS equation is not integrable.

The Discrete Nonlinear Schrödinger (DNLS) equation

is not integrable

- On the contrary, the DNLS equation is not integrable.
- Exact solutions must be found numerically. The most common methods are based on the anti-continuous limit (R.S. MacKay and S. Aubry. Nonlinearity 7 (1994) 1263)

The Discrete Nonlinear Schrödinger (DNLS) equation

is not integrable

- On the contrary, the DNLS equation is not integrable.
- Exact solutions must be found numerically. The most common methods are based on the anti-continuous limit (R.S. MacKay and S. Aubry. Nonlinearity 7 (1994) 1263)
- Stationary solutions, $\psi_n(t) = \mathrm{e}^{i\omega t}\phi_n$, are determined by the equation system:

$$\omega\phi_n = \phi_n^3 + (\phi_{n+1} + \phi_{n-1} - 2\phi_n)$$

The Discrete Nonlinear Schrödinger (DNLS) equation

is not integrable

- On the contrary, the DNLS equation is not integrable.
- Exact solutions must be found numerically. The most common methods are based on the anti-continuous limit (R.S. MacKay and S. Aubry. Nonlinearity 7 (1994) 1263)
- Stationary solutions, $\psi_n(t) = \mathrm{e}^{i\omega t}\phi_n$, are determined by the equation system:

$$\omega\phi_n = \phi_n^3 + (\phi_{n+1} + \phi_{n-1} - 2\phi_n)$$

- The aim of this talk is to show

how can ϕ_n be approximated for discrete solitons

Summary

Discrete solitons

Sievers–Takeno and Page modes

- Discrete solitons are localized solutions of the DNLS equation.

Discrete solitons

Sievers–Takeno and Page modes

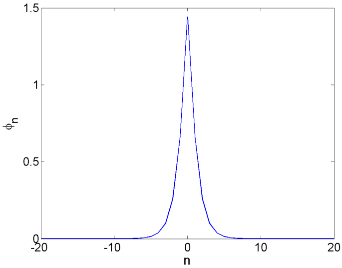
- Discrete solitons are localized solutions of the DNLS equation.
- We focus on site-centered (Sievers–Takeno) and bond-centered (Page) modes.

Discrete solitons

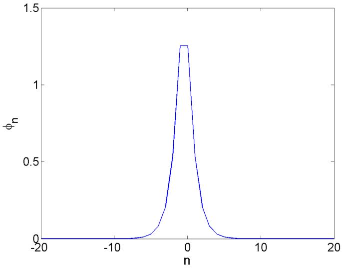
Sievers–Takeno and Page modes

- Discrete solitons are localized solutions of the DNLS equation.
- We focus on site-centered (Sievers–Takeno) and bond-centered (Page) modes.
- Exact profiles (numerical calculations):

Sievers–Takeno (ST) mode



Page (P) mode



Discrete solitons

A first approximation

- Kivshar and Campbell made the first attempt to find the profiles (Yu.S. Kivshar and D.K. Campbell. PRE 48 (1993) 3077)

Discrete solitons

A first approximation

- Kivshar and Campbell made the first attempt to find the profiles (Yu.S. Kivshar and D.K. Campbell. PRE 48 (1993) 3077)
- They supposed that

$$\phi_n = \begin{cases} A_1 \text{ if } n = 0 \\ \chi_1 \text{ if } |n| = 1 \\ 0 \text{ otherwise} \end{cases} \qquad \phi_n = \begin{cases} A_2 \text{ if } n = -1, 0 \\ \chi_2 \text{ if } n = -2, 1 \\ 0 \text{ otherwise} \end{cases}$$

ST mode P mode

Discrete solitons

A first approximation

- Kivshar and Campbell made the first attempt to find the profiles (Yu.S. Kivshar and D.K. Campbell. PRE 48 (1993) 3077)
- They supposed that

$$\phi_n = \begin{cases} A_1 \text{ if } n = 0 \\ \chi_1 \text{ if } |n| = 1 \\ 0 \text{ otherwise} \end{cases} \qquad \phi_n = \begin{cases} A_2 \text{ if } n = -1, 0 \\ \chi_2 \text{ if } n = -2, 1 \\ 0 \text{ otherwise} \end{cases}$$

ST mode P mode

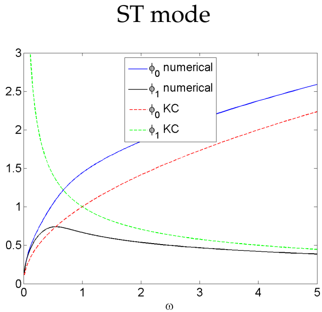
- After introduction in the DNLS equation, the values of A_1, A_2, χ_1 and χ_2, are found:

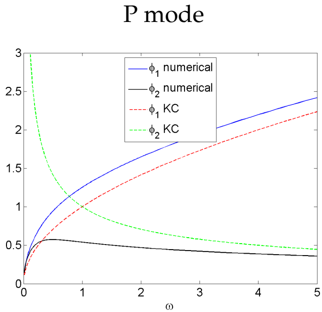
$$A_{1,2} = \sqrt{\omega}, \qquad \chi_{1,2} = \omega^{-1/2}$$

Discrete solitons

A first approximation

- Comparison with numerical profiles.





Discrete solitons

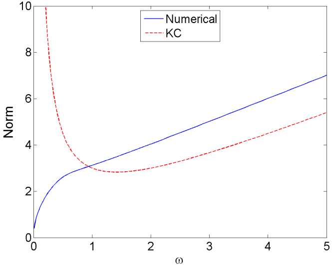
A first approximation

- Comparison of the norm, $N = \sum_n |\phi_n|^2$.

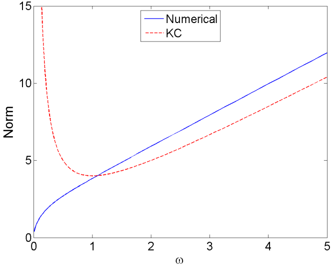
Discrete solitons
A first approximation

- Comparison of the norm, $N = \sum_n |\phi_n|^2$.
- In the KC approximation, $N = \omega + 2/\omega$ for the ST mode, and $N = 2\omega + 2/\omega$ for the P mode.

ST mode



P mode



Summary

Variational approach

The effective Hamiltonian

- A further approximation is to suppose that ϕ_n has a peaked profile:

ST mode	P mode
$\phi_n = A_1 \exp(-\alpha_1 \lvert n \rvert)$	$\phi_n = A_2 \exp(-\alpha_2 \lvert n \rvert)$

Variational approach

The effective Hamiltonian

- A further approximation is to suppose that ϕ_n has a peaked profile:

ST mode	P mode
$\phi_n = A_1 \exp(-\alpha_1 \lvert n \rvert)$	$\phi_n = A_2 \exp(-\alpha_2 \lvert n \rvert)$

- The values of A and α are determined through a variational approach. To this end, a Hamiltonian must be minimized (B.A. Malomed and M.I. Weinstein. Phys Lett A 220 (1996) 91)

Variational approach

The effective Hamiltonian

- A further approximation is to suppose that ϕ_n has a peaked profile:

$$\begin{array}{cc} \underline{\text{ST mode}} & \underline{\text{P mode}} \\ \phi_n = A_1 \exp(-\alpha_1 |n|) & \phi_n = A_2 \exp(-\alpha_2 |n|) \end{array}$$

- The values of A and α are determined through a variational approach. To this end, a Hamiltonian must be minimized (B.A. Malomed and M.I. Weinstein. Phys Lett A 220 (1996) 91)
- The DNLS Hamiltonian is given by:

$$H = -\sum_n \left[\frac{1}{2} |\psi_n - \psi_{n-1}|^2 + \frac{1}{4} |\psi_n|^4 \right]$$

Variational approach

The effective Hamiltonian

- An effective Hamiltonian is found introducing the previous ansätze:

Variational approach

The effective Hamiltonian

- An effective Hamiltonian is found introducing the previous ansätze:
  - ST mode:

$$H_{\text{eff}} = 2N_2(1 - \operatorname{sech}\alpha_1) - N_2^2 \frac{\tanh^2\alpha_1}{2\tanh 2\alpha_1}, \qquad N_1 = A_1^2 \coth\alpha_1$$

Variational approach

The effective Hamiltonian

- An effective Hamiltonian is found introducing the previous ansätze:
    - ST mode:

$$H_{\text{eff}} = 2N_2(1 - \operatorname{sech}\alpha_1) - N_2^2 \frac{\tanh^2\alpha_1}{2\tanh 2\alpha_1}, \qquad N_1 = A_1^2 \coth\alpha_1$$

    - P mode:

$$H_{\text{eff}} = 2N_2 \left(\frac{\cosh\alpha_2 - 1}{\sinh\alpha_2 + \cosh\alpha_2} \right) - \frac{1}{4} N_2^2 \tanh\alpha_2, \qquad N_2 = A_2^2 \operatorname{csch}\alpha_2$$

Variational approach

The effective Hamiltonian

- An effective Hamiltonian is found introducing the previous ansätze:
  - ST mode:

$$H_{\text{eff}} = 2N_2(1 - \operatorname{sech}\alpha_1) - N_2^2 \frac{\tanh^2\alpha_1}{2\tanh 2\alpha_1}, \qquad N_1 = A_1^2 \coth\alpha_1$$

  - P mode:

$$H_{\text{eff}} = 2N_2\left(\frac{\cosh\alpha_2 - 1}{\sinh\alpha_2 + \cosh\alpha_2}\right) - \frac{1}{4}N_2^2\tanh\alpha_2, \qquad N_2 = A_2^2 \operatorname{csch}\alpha_2$$

- The Hamiltonian is minimized with respect to α, so that $N(\alpha)$ is found:

$$N_1 = \frac{4\cosh\alpha_1 \sinh^2 2\alpha_1}{\sinh 4\alpha_1 - \sinh 2\alpha_1}, \qquad N_2 = \frac{8(1 - \cosh\alpha_2 + \sinh\alpha_2)\cosh^2\alpha_2}{\sinh\alpha_2 + \cosh\alpha_2}$$

Variational approach

The effective Hamiltonian

- An extra equation $\alpha(\omega)$ is needed. It is done by a tail analysis.

Variational approach

The effective Hamiltonian

- An extra equation $\alpha(\omega)$ is needed. It is done by a tail analysis.
- In the tail, amplitudes are small. Soliton tails can be solutions of the linearized equation:

$$\omega\phi_n = (\phi_{n+1} + \phi_{n-1} - 2\phi_n)$$

Variational approach

The effective Hamiltonian

- An extra equation $\alpha(\omega)$ is needed. It is done by a tail analysis.
- In the tail, amplitudes are small. Soliton tails can be solutions of the linearized equation:

$$\omega\phi_n = (\phi_{n+1} + \phi_{n-1} - 2\phi_n)$$

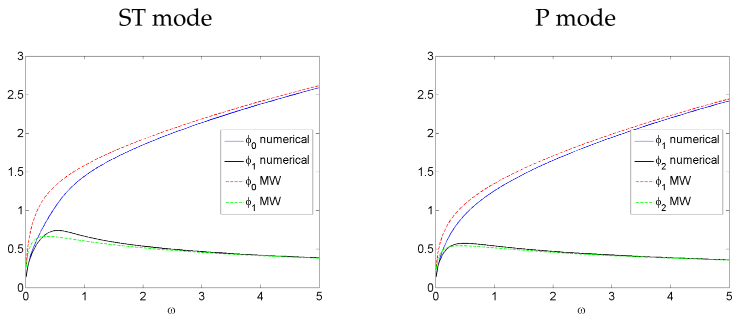
- Thus,

$$\cosh\alpha = 1 + \frac{\omega}{2}$$

for both ST and P modes.

Malomed and Weinstein Variational approach

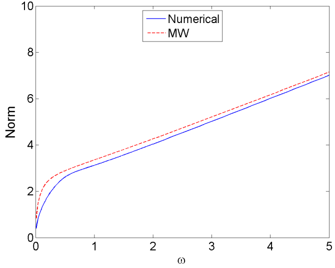
Comparison with numerical profiles



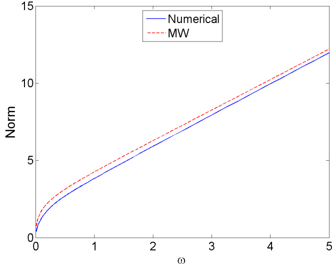
Malomed and Weinstein Variational approach

Comparison of the norm

ST mode



P mode



Summary

Variational approach

The effective Lagragian

- A further approximation is to minimize an effective Lagragian with respect to α and N.

Variational approach

The effective Lagragian

- A further approximation is to minimize an effective Lagragian with respect to α and N.
- The DNLS Lagragian is given by:

$$L = \frac{1}{2}\sum_n \left[\mathrm{i}\left(\psi_n^* \dot{\psi}_n - \psi_n \dot{\psi}_n^*\right) + |\psi_n - \psi_{n-1}|^2 + \frac{1}{2}|\psi_n|^4 \right]$$

Variational approach

The effective Lagragian

- A further approximation is to minimize an effective Lagragian with respect to α and N.
- The DNLS Lagragian is given by:

$$L = \frac{1}{2}\sum_n \left[\mathrm{i}\left(\psi_n^* \dot{\psi}_n - \psi_n \dot{\psi}_n^*\right) + |\psi_n - \psi_{n-1}|^2 + \frac{1}{2}|\psi_n|^4 \right]$$

- Substituting the peaked ansätze, the effective Lagragians are found:

Variational approach

The effective Lagragian

- A further approximation is to minimize an effective Lagragian with respect to α and N.
- The DNLS Lagragian is given by:

$$L = \frac{1}{2} \sum_n \left[\mathrm{i} \left(\psi_n^* \dot{\psi}_n - \psi_n \dot{\psi}_n^* \right) + |\psi_n - \psi_{n-1}|^2 + \frac{1}{2} |\psi_n|^4 \right]$$

- Substituting the peaked ansätze, the effective Lagragians are found:
  - ST mode:

$$L_{\mathrm{eff}} = N_1 (2 \operatorname{sech} \alpha_1 - \omega - 2) + N_1^2 \frac{\tanh^2 \alpha_1}{2 \tanh 2\alpha_1}, \quad N_1 = A_1^2 \coth \alpha_1$$

Variational approach

The effective Lagragian

- A further approximation is to minimize an effective Lagragian with respect to α and N.
- The DNLS Lagragian is given by:

$$L = \frac{1}{2}\sum_n \left[\mathrm{i}\left(\psi_n^* \dot{\psi}_n - \psi_n \dot{\psi}_n^*\right) + |\psi_n - \psi_{n-1}|^2 + \frac{1}{2}|\psi_n|^4 \right]$$

- Substituting the peaked ansätze, the effective Lagragians are found:
  - ST mode:

$$L_{\text{eff}} = N_1(2\operatorname{sech}\alpha_1 - \omega - 2) + N_1^2 \frac{\tanh^2\alpha_1}{2\tanh 2\alpha_1}, \quad N_1 = A_1^2 \coth\alpha_1$$

  - P mode:

$$L_{\text{eff}} = N_2\left(\frac{2(1-\cosh\alpha_2)}{\sinh\alpha_2 + \cosh\alpha_2} - \omega\right) + \frac{N_2^2}{4}\tanh\alpha_2, \quad N_2 = A_2^2 \operatorname{csch}\alpha_2$$

Variational approach

The effective Lagragian

- The Lagragian is minimized with respect to α, so that $N(\alpha)$ is found:

$$N_1 = \frac{4\cosh\alpha_1 \sinh^2 2\alpha_1}{\sinh 4\alpha_1 - \sinh 2\alpha_1}, \quad N_2 = \frac{8(1-\cosh\alpha_2 + \sinh\alpha_2)\cosh^2\alpha_2}{\sinh\alpha_2 + \cosh\alpha_2}$$

which are the same values obtained minimizing the Hamiltonian.

Variational approach

The effective Lagragian

- The Lagragian is minimized with respect to α, so that $N(\alpha)$ is found:

$$N_1 = \frac{4\cosh\alpha_1 \sinh^2 2\alpha_1}{\sinh 4\alpha_1 - \sinh 2\alpha_1}, \quad N_2 = \frac{8(1-\cosh\alpha_2+\sinh\alpha_2)\cosh^2\alpha_2}{\sinh\alpha_2+\cosh\alpha_2}$$

  which are the same values obtained minimizing the Hamiltonian.

- The relation $\omega(\alpha)$ is found upon minimization of the Lagragian with respect to N:

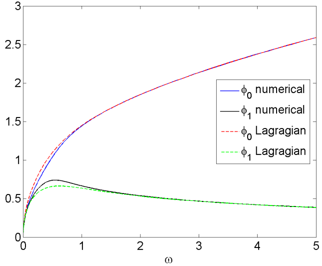
$$\omega = 2(\operatorname{sech}\alpha_1 - 1) + N_1 \frac{\tanh^2\alpha_1}{\tanh 2\alpha_1} \quad \text{(ST mode)}$$

$$\omega = \frac{2(1-\cosh\alpha_2)}{\sinh\alpha_2+\cosh\alpha_2} + \frac{1}{2} N_2 \tanh\alpha_2 \quad \text{(P mode)}$$

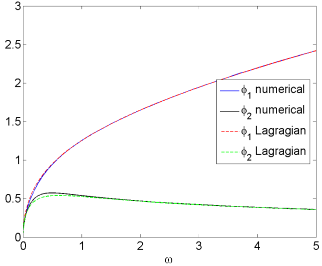
Lagragian approach

Comparison with numerical profiles

ST mode

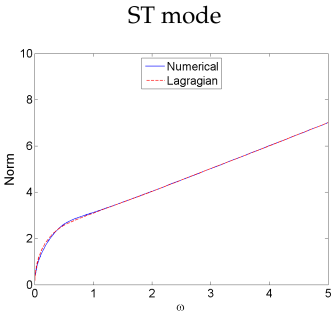


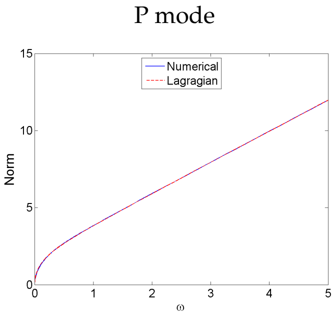
P mode



Lagragian approach

Comparison of the norm





Summary

The DNLS map

and the saddle-point

- The stationary DNLS can be written as a 2-D real map, making the following change of variable:

$$y_n = \phi_n, \qquad x_n = \phi_{n-1}$$

so that

$$\begin{cases} x_{n+1} = y_n \\ y_{n+1} = -y_n^3 + (\omega + 2)y_n - x_n \end{cases}$$

The DNLS map
and the saddle-point

- The stationary DNLS can be written as a 2-D real map, making the following change of variable:

$$y_n = \phi_n, \qquad x_n = \phi_{n-1}$$

so that

$$\begin{cases} x_{n+1} = y_n \\ y_{n+1} = -y_n^3 + (\omega + 2)y_n - x_n \end{cases}$$

- This map has a saddle-point at $x_n = y_n = 0$. In consequence, there are a 1-D stable and a 1-D unstable manifold.

The DNLS map

and the homoclinic tangle

- The stable and unstable manifolds intersect and form a homoclinic tangle. Each of the intersections correspond to a localized solution.
  (D. Hennig *et al*. PRE 54 (1996) 5788)
  T. Bountis *et al*. Phys Lett A 268 (2000) 50
  G. L. Alfimov *et al*. Physica D 194 (2004) 127)

The DNLS map

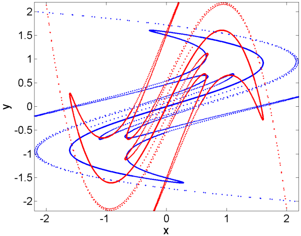
and the homoclinic tangle

- The stable and unstable manifolds intersect and form a homoclinic tangle. Each of the intersections correspond to a localized solution.
  (D. Hennig *et al*. PRE 54 (1996) 5788)
  T. Bountis *et al*. Phys Lett A 268 (2000) 50
  G. L. Alfimov *et al*. Physica D 194 (2004) 127)
- These intersection points corresponds to (ϕ_{-1}, ϕ_0). $\phi_{n>2}$ are determined upon application of the map.

The DNLS map

and the homoclinic tangle

- The stable and unstable manifolds intersect and form a homoclinic tangle. Each of the intersections correspond to a localized solution.
  (D. Hennig *et al*. PRE 54 (1996) 5788)
  T. Bountis *et al*. Phys Lett A 268 (2000) 50
  G. L. Alfimov *et al*. Physica D 194 (2004) 127)
- These intersection points corresponds to (ϕ_{-1}, ϕ_0). $\phi_{n>2}$ are determined upon application of the map.
- The tangle is symmetric with respect to the lines $y = x$ and $y = -x$.



The DNLS map

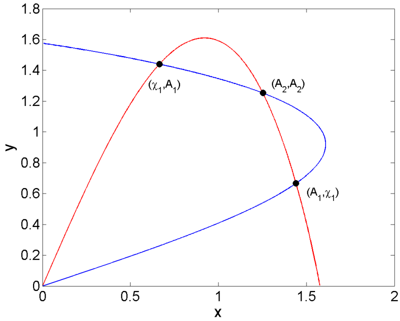
ST and P modes

- The ST and P modes are due to the intersection of the first homoclinic windings.

The DNLS map

ST and P modes

- The ST and P modes are due to the intersection of the first homoclinic windings.
- The homoclinic orbit considered for those modes is substantially simplified.



Summary

The approximated unstable manifold

A third order polynomial

- The unstable manifold can be approximated by a third order polynomial:
$$y = \lambda x - x^3$$

The approximated unstable manifold

A third order polynomial

- The unstable manifold can be approximated by a third order polynomial:

$$y = \lambda x - x^3$$

- λ is the eigenvalue of the linearized DNLS map fulfilling that $\lambda > 1$:

$$\lambda = \frac{(2+\omega) + \sqrt{\omega(\omega+4)}}{2}$$

The approximated unstable manifold

A third order polynomial

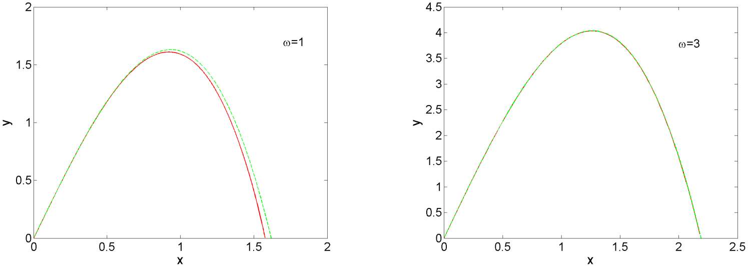
- The unstable manifold can be approximated by a third order polynomial:

$$y = \lambda x - x^3$$

- λ is the eigenvalue of the linearized DNLS map fulfilling that $\lambda > 1$:

$$\lambda = \frac{(2+\omega) + \sqrt{\omega(\omega+4)}}{2}$$

- Comparison with the exact unstable manifold:



The approximated unstable and stable manifolds

Intersections

- The unstable manifold can be transformed into the stable manifold through the change $x \leftrightarrow y$. Thus, the stable manifold is given by:

$$x = \lambda y - y^3$$

The approximated unstable and stable manifolds

Intersections

- The unstable manifold can be transformed into the stable manifold through the change $x \leftrightarrow y$. Thus, the stable manifold is given by:

$$x = \lambda y - y^3$$

- Approximate solutions for the ST and P modes are given by the intersections of both manifolds, leading to the equation:

$$x = \lambda(\lambda x - x^3) - (\lambda x - x^3)^3$$

The approximated unstable and stable manifolds

Intersections

- The unstable manifold can be transformed into the stable manifold through the change $x \leftrightarrow y$. Thus, the stable manifold is given by:

$$x = \lambda y - y^3$$

- Approximate solutions for the ST and P modes are given by the intersections of both manifolds, leading to the equation:

$$x = \lambda(\lambda x - x^3) - (\lambda x - x^3)^3$$

- Solving this equation, we are able to find A_1, χ_1 and A_2. χ_2 is found by application of the map.

The approximated unstable and stable manifolds

Intersections

- For the ST mode,

$$A_1 = \sqrt{\frac{\lambda + \sqrt{\lambda^2 - 4}}{2}}$$

$$\chi_1 = \sqrt{\frac{\lambda - \sqrt{\lambda^2 - 4}}{2}}$$

The approximated unstable and stable manifolds

Intersections

- For the ST mode,

$$A_1 = \sqrt{\frac{\lambda + \sqrt{\lambda^2 - 4}}{2}}$$

$$\chi_1 = \sqrt{\frac{\lambda - \sqrt{\lambda^2 - 4}}{2}}$$

- For the P mode,

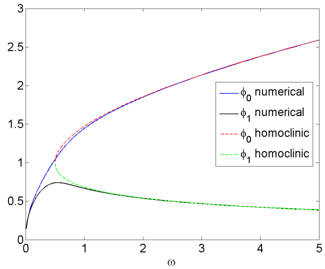
$$A_2 = \sqrt{\lambda - 1}$$

$$\chi_2 = (\lambda + \omega - 2)\sqrt{\lambda - 1}$$

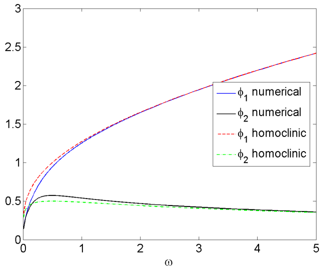
The approximated unstable and stable manifolds

Comparison with numerical profiles

ST mode



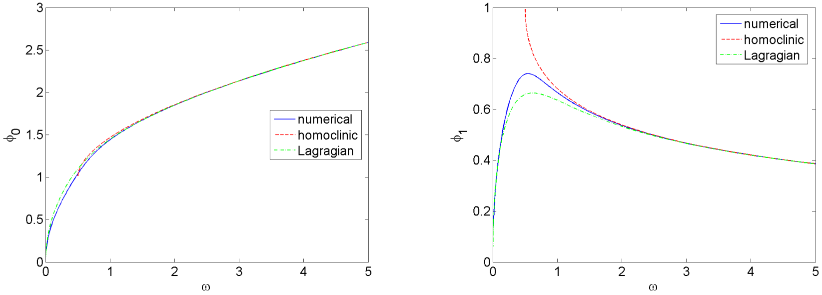
P mode



- For the ST mode, there are only intersections if $\omega > 1/2$

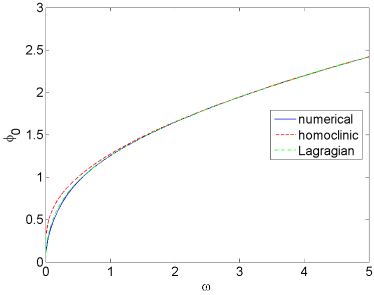
Variational and homoclinic approximations

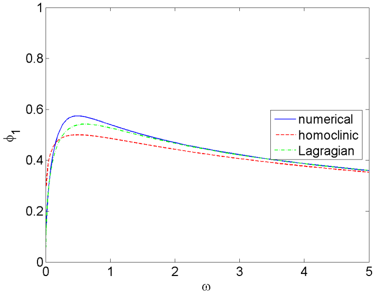
ST mode. Comparison with numerical profiles



Variational and homoclinic approximations

P mode. Comparison with numerical profiles





Conclusions

- The Variational Approach behaves better than the approximation to the Homoclinic tangle.

Conclusions

- The Variational Approach behaves better than the approximation to the Homoclinic tangle.
- Furthermore, the Homoclinic approximation cannot be used for calculating the norm.

Conclusions

- The Variational Approach behaves better than the approximation to the Homoclinic tangle.
- Furthermore, the Homoclinic approximation cannot be used for calculating the norm.
- However, the Homoclinic approximation leads to simpler expressions for the profiles.

Conclusions

- The Variational Approach behaves better than the approximation to the Homoclinic tangle.
- Furthermore, the Homoclinic approximation cannot be used for calculating the norm.
- However, the Homoclinic approximation leads to simpler expressions for the profiles.
- None of the approaches fit for $\omega \to 0$.

Conclusions

- The Variational Approach behaves better than the approximation to the Homoclinic tangle.
- Furthermore, the Homoclinic approximation cannot be used for calculating the norm.
- However, the Homoclinic approximation leads to simpler expressions for the profiles.
- None of the approaches fit for $\omega \to 0$.
- Possible explanations:

Conclusions

- The Variational Approach behaves better than the approximation to the Homoclinic tangle.
- Furthermore, the Homoclinic approximation cannot be used for calculating the norm.
- However, the Homoclinic approximation leads to simpler expressions for the profiles.
- None of the approaches fit for $\omega \to 0$.
- Possible explanations:
  - The profile is Gaussian for small ω. This should be take into account.

Conclusions

- The Variational Approach behaves better than the approximation to the Homoclinic tangle.
- Furthermore, the Homoclinic approximation cannot be used for calculating the norm.
- However, the Homoclinic approximation leads to simpler expressions for the profiles.
- None of the approaches fit for $\omega \to 0$.
- Possible explanations:
  - The profile is Gaussian for small ω. This should be take into account.
  - The unstable manifold can be approximated by a 5th order polynomial (but, a 10th degree equation must be solved)

Conclusions

- Thank you for your attention!
- More information:
  - Nonlinear Physics Group (GFNL) of Seville University:

    `http://www.grupo.us.es/gfnl`
  - My personal web page

    `http://www.personal.us.es/jcuevas`