Bifurcation Analysis of Discrete Breathers in a Nonlinear bent chain of oscillators

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Outline

- Concept of discrete breather.
- Discrete breathers in homogeneous lattices. Bifurcations.
- Discrete breathers in bent chains. Bifurcations.
- Conclusions.

Discrete breathers

MOVIE

Discrete breathers

- Different frameworks:
  - Discrete Nonlinear Klein–Gordon Equation:

$$\ddot{u}_n + V'(u_n) + C \sum_m W'(u_n, u_m) = 0$$

  - Fermi–Pasta–Ulam model: $V(u_n) = 0 \; \forall n$.
  - Discrete Nonlinear Schrödinger Equation:

$$i\dot{u}_n + \gamma |u_n|^2 u_n - C(2u_n - u_{n+1} - u_{n-1}) = 0$$

- $u_n \in \mathcal{E}_s^2(\omega_{\mathrm{b}})$; $V, W \in \mathcal{C}^2$; $n \in \mathbb{Z}^d$.

MacKay-Aubry theorem

- R.S. MacKay and S. Aubry. Nonlinearity **7** (1994) 1623-1643.
- *If a periodic orbit of* $H = p^2/2 + V(u)$ *with action* I_0 *is non-resonant and anharmonic, then the periodic orbit of* $\ddot{u}_n + V'(u_n) + C(2u_n - u_{n+1} - u_{n-1}) = 0$ *at* $C = 0$ *given by* $x_0(t) = X(I_0, \omega_\mathrm{b}(I_0)t)$ *and* $x_n(t) = 0$ *for all sites* $n \neq 0$ *has a locally unique continuation as a periodic orbit of* $\ddot{u}_n + V'(u_n) + C(2u_n - u_{n+1} - u_{n-1}) = 0$ *of the same period* $T = 2\pi/\omega_\mathrm{b}(I_0)$ *for* C *small enough.*
- *The* C^1 *norm of the oscillation on site* n *for that periodic solution decays exponentially as* $n \to \infty$, *as long as* DF *remains invertible.*

Linear stability

- Dynamical equations of a perturbation $u_n \to u_n + \xi_n$ ($\xi \in \mathcal{C}^2$):

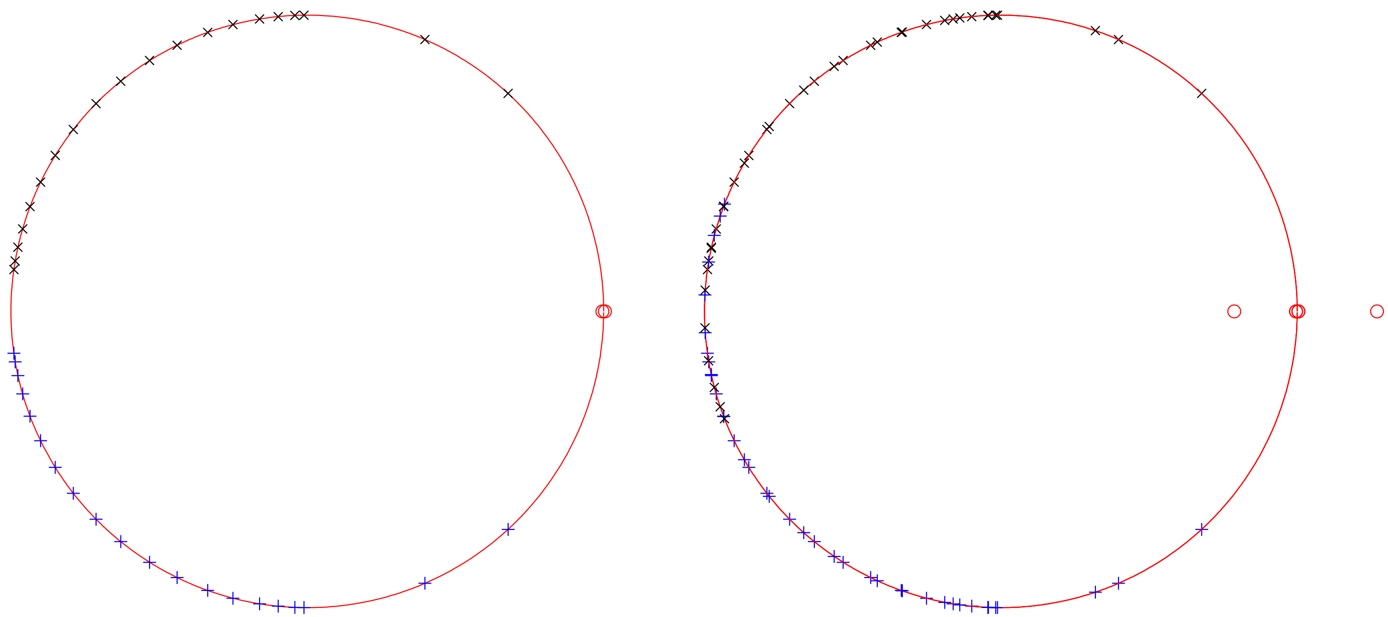
$$\ddot{\xi}_n(t) + V''(u_n(t))\xi_n(t) - C\sum_i W''(u_{n+i}(t) - u_n(t))(\xi_{n+i}(t) - \xi_n(t)) +$$
$$+C\sum_i W''(u_n(t) - u_{n-i}(t))(\xi_n(t) - \xi_{n-i}(t)) = 0$$

- Linear stability $\to$ Floquet analysis:

$$\begin{bmatrix} \xi(T) \\ \dot{\xi}(T) \end{bmatrix} = \mathcal{F}_o \begin{bmatrix} \xi(0) \\ \dot{\xi}(0) \end{bmatrix}$$

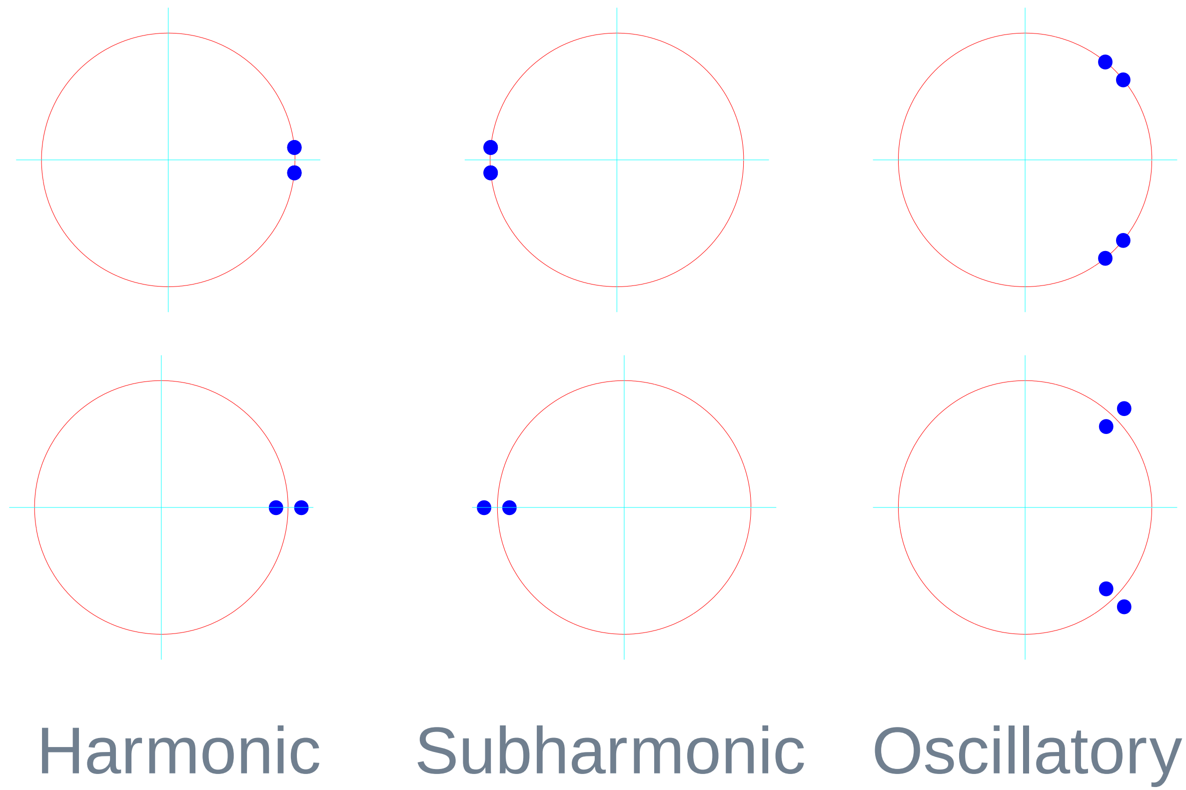
- Floquet's operator is real and symplectic:
  - λ, $1/\lambda$, λ^*, $1/\lambda^*$ are eigenvalues.
  - A discrete breather in a Hamiltonian Klein–Gordon lattice is linear stable if and only if all eigenvalues are on the unit circle.

Linear stability

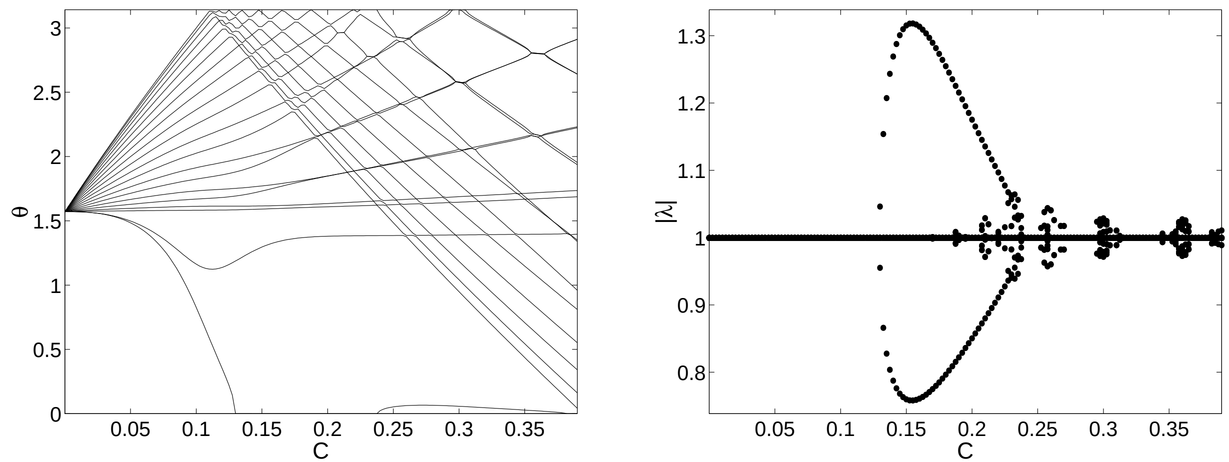


Stable | Unstable

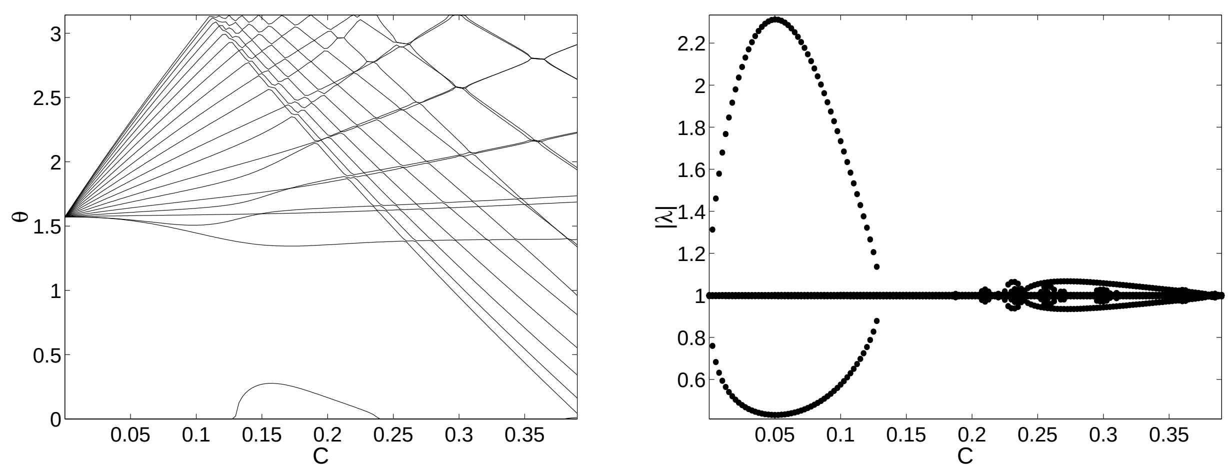
Bifurcations



Stability exchange (Marginal mode)



Site-centered breather



Bond-centered breather

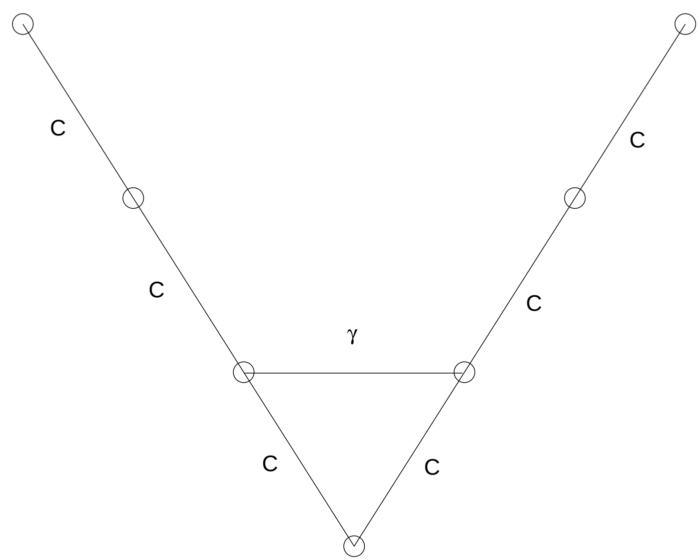
Mobile breather

MOVIE

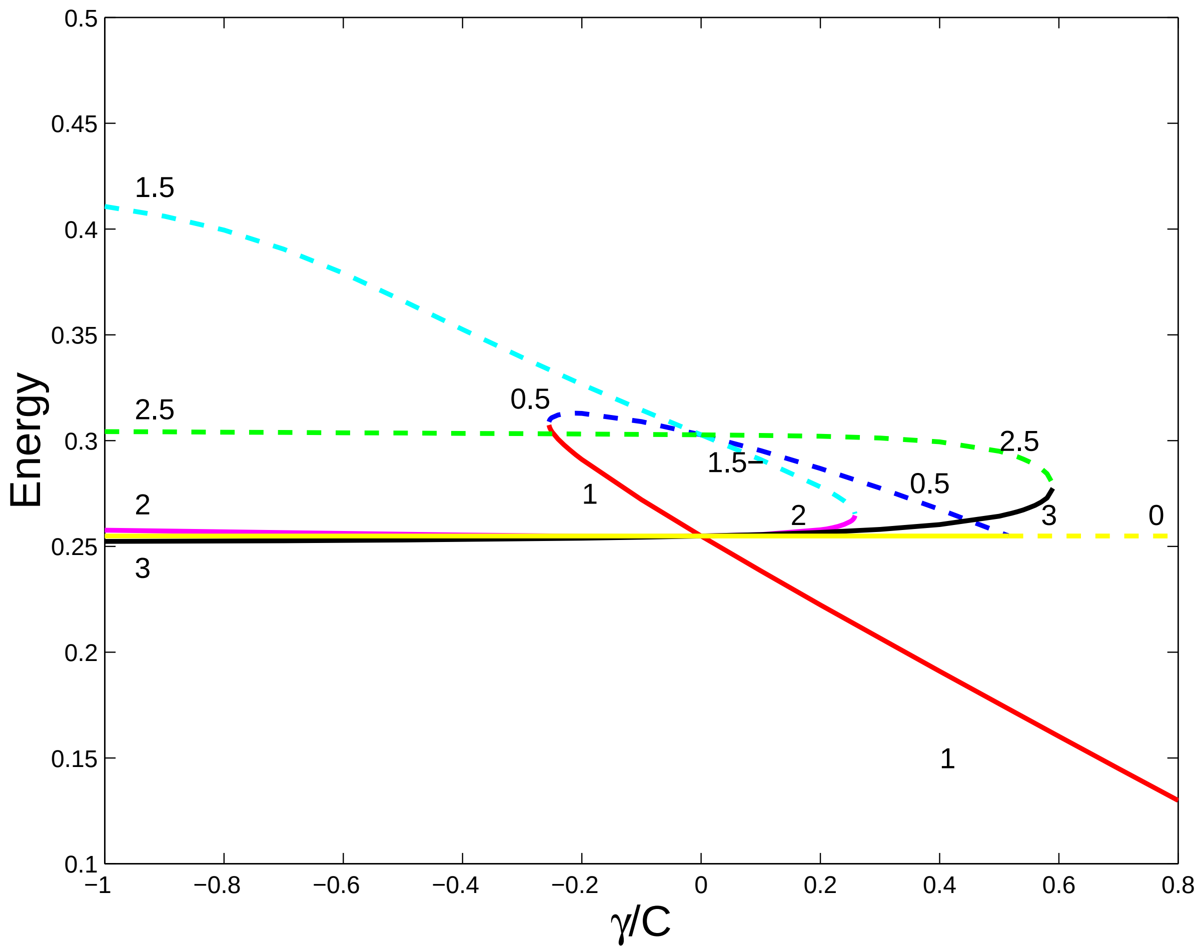
Bent (wedged) chain

$$\ddot{u}_n + V'(u_n) + C\sum_m (2u_n - u_{n+1} - u_{n-1}) + \gamma[(u_n - u_{n-2})\delta_{n,1} + (u_n - u_{n+2})\delta_{n,-1}] = 0$$

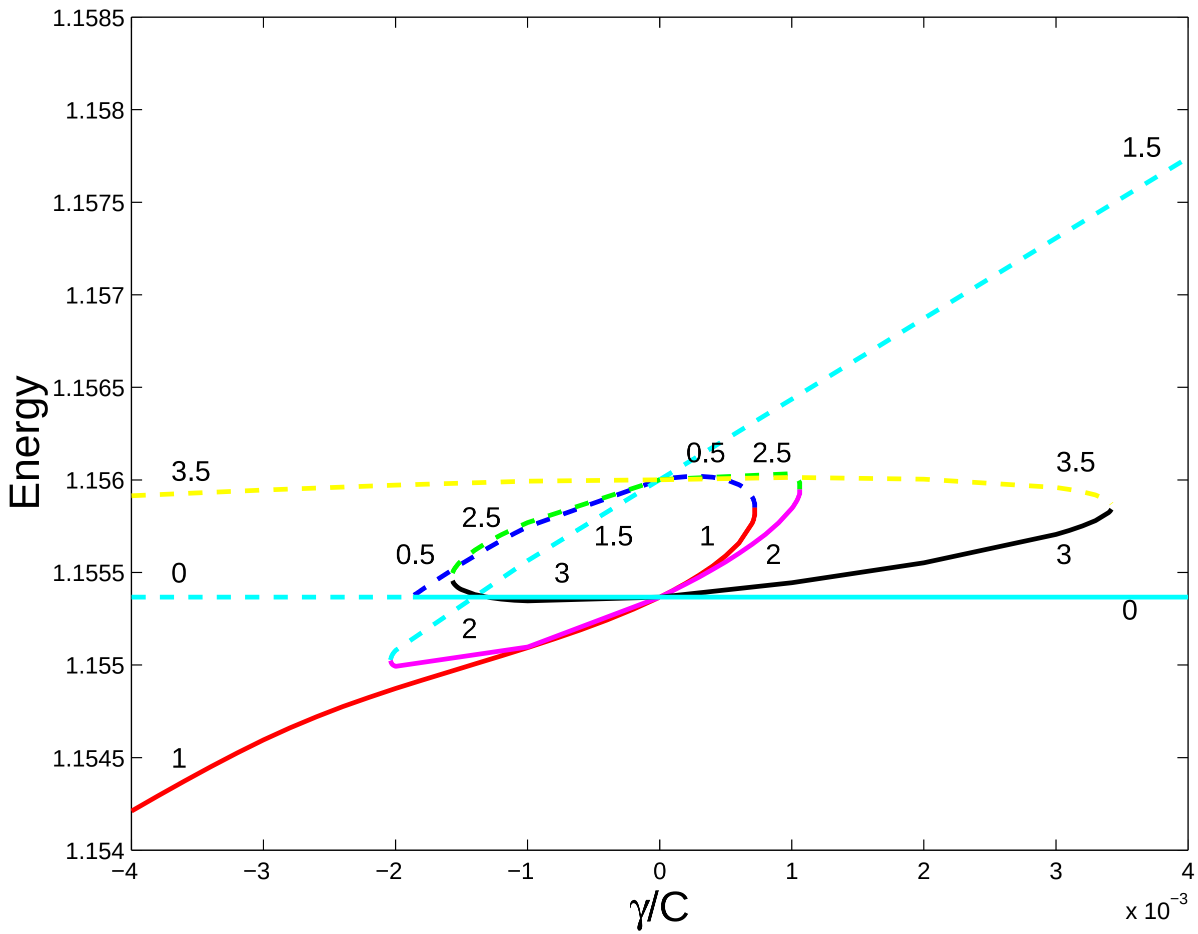
- u_n: Out-of-plane displacements.
- $n = 0$: Vertex of the wedge.



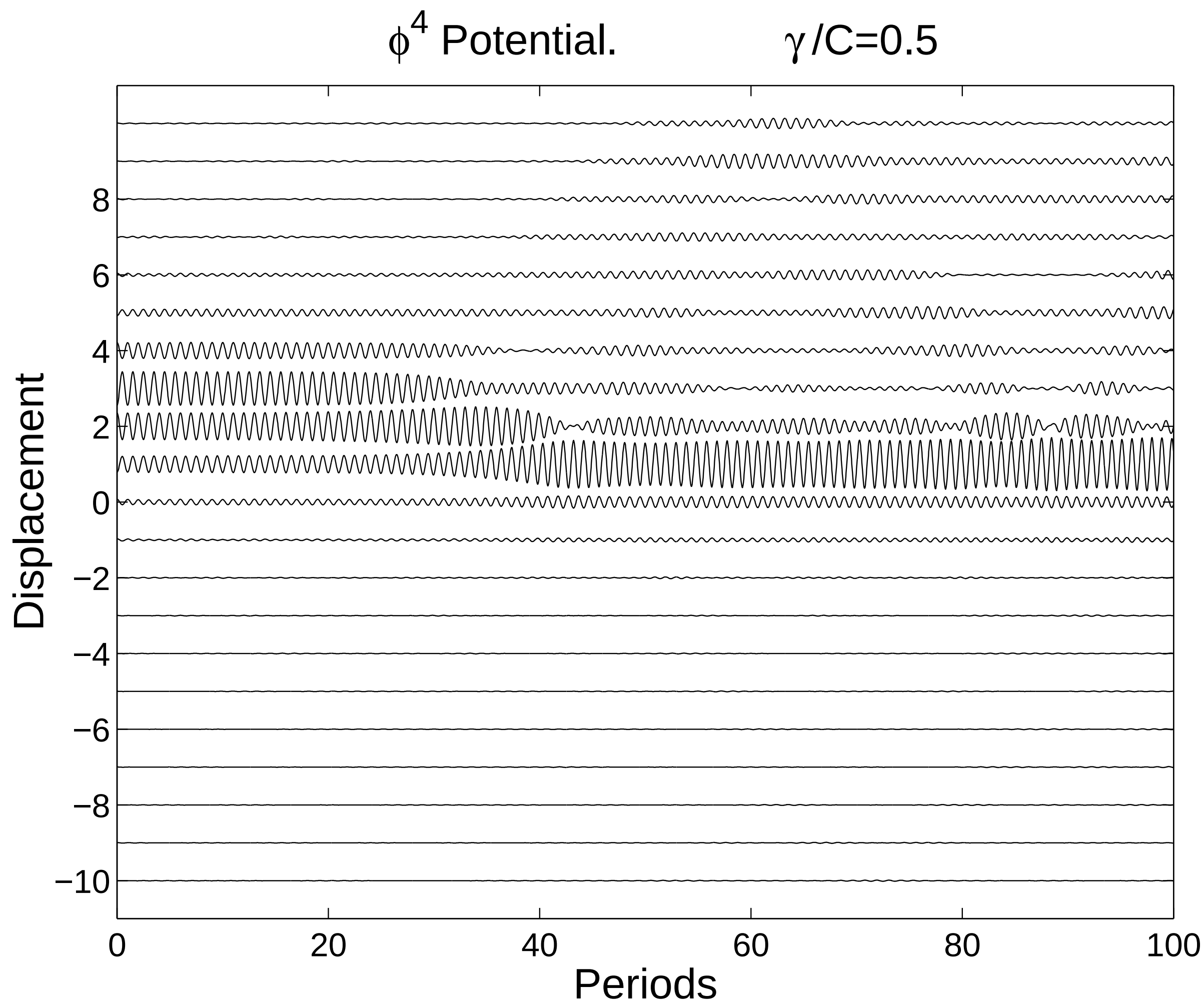
Bifurcations. ϕ^4 potential



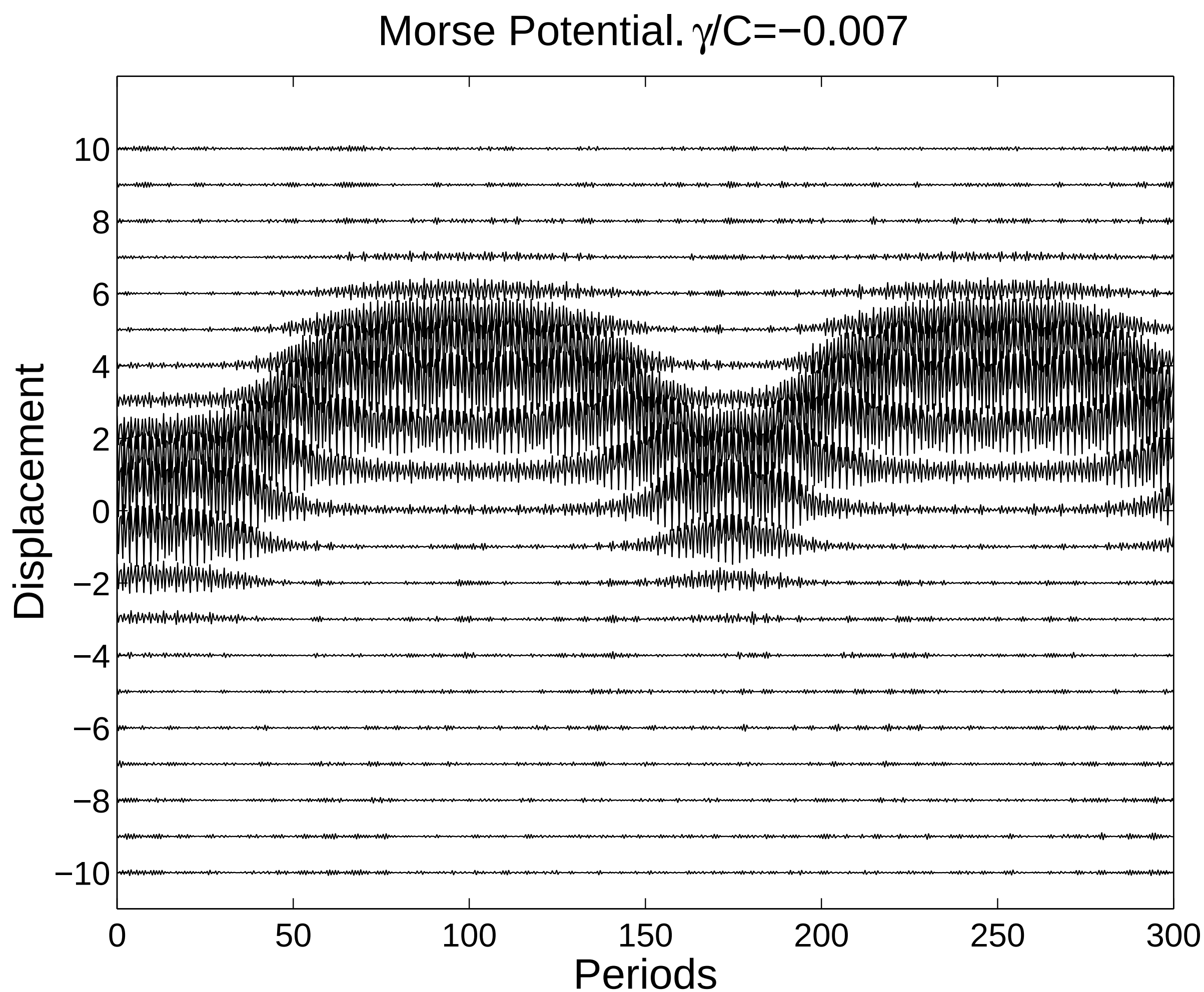
Bifurcations. Morse potential



Switching in ϕ^4 potential



Mobility in Morse potential



Conclusions

- Homogeneous Klein–Gordon lattices: Summary of (stability) bifurcations.
- Wedged Klein–Gordon chains: Ground state changes with γ.
  - ϕ^4 potential: Switching.
  - Morse potential: Spontaneous mobility.
- Nonlinear Physics Group website:

http://www.us.es/gfnl