

Engineered Nonlinear Excitations in Magnetic Nanostructures

***Importance of internal shape mode in
magnetic vortex dynamics***

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Outline

- Model
- Magnetic **nanodots**
- Vortex dynamics: switching of vortex polarization by magnetic fields
- Vortex dynamics: circular limit cycles
- Important role of **internal degrees of freedom**
- Conclusions

Model

Heisenberg Hamiltonian for classical spins $\vec{S}_{\vec{n}}$ located at sites \vec{n} of a square lattice plus a Zeeman interaction with the IP rotating field

$$\mathcal{H} = -\frac{J}{2} \sum_{(\vec{n}, \vec{a})} \left[\vec{S}_{\vec{n}} \cdot \vec{S}_{\vec{n}+\vec{a}} - \delta S_{\vec{n}}^z S_{\vec{n}+\vec{a}}^z \right] - \gamma B \sum_{\vec{n}} \left[S_{\vec{n}}^x \cos(\omega t) + S_{\vec{n}}^y \sin(\omega t) \right]$$

Dynamics

Landau-Lifshitz equations:

$$\frac{d\vec{S}_{\vec{n}}}{dt} = -\vec{S}_{\vec{n}} \times \frac{\partial \mathcal{H}}{\partial \vec{S}_{\vec{n}}} - \epsilon \frac{\vec{S}_{\vec{n}}}{S} \times \frac{d\vec{S}_{\vec{n}}}{dt}$$

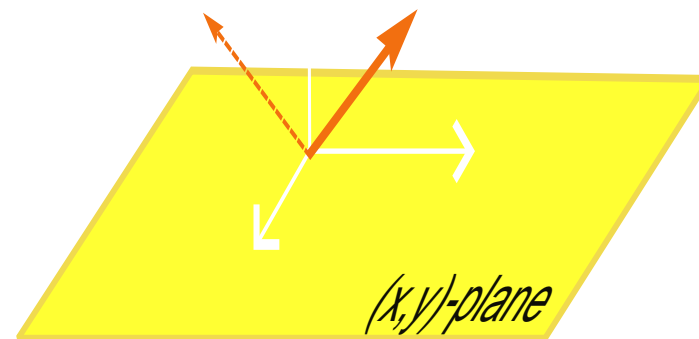
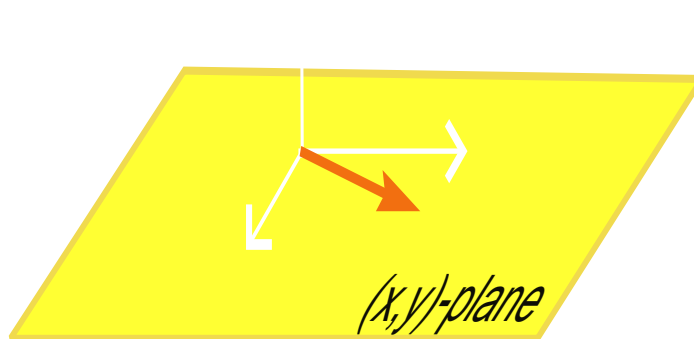
ϵ is the damping coefficient.

Ground State

$$S_{\vec{n}}^x = \sqrt{1 - m_{\vec{n}}^2} \cos \phi_{\vec{n}}, \quad S_{\vec{n}}^y = \sqrt{1 - m_{\vec{n}}^2} \sin \phi_{\vec{n}},$$

$$S_{\vec{n}}^z = m_{\vec{n}}$$

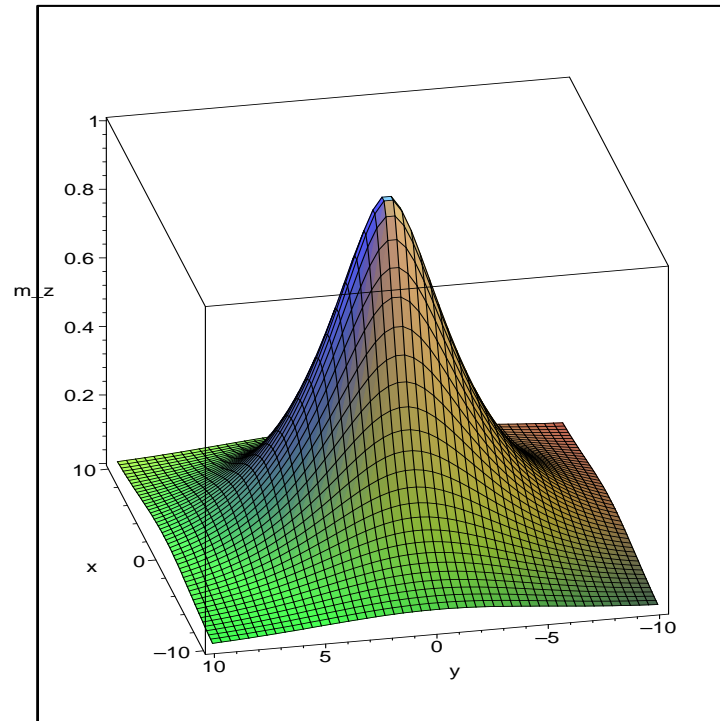
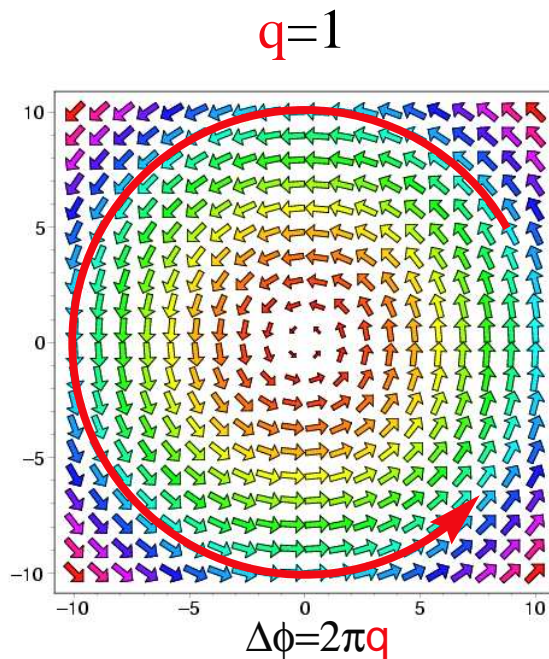
Without field: $m = 0$, $\phi = \text{const}$ With field: $m \approx \omega$,
 $\phi = \omega t$



Vortex structure

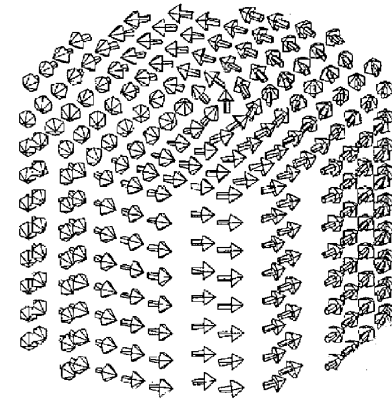
$$\phi(x, y) = \varphi_0 + q \arctan(y/x).$$

$$m(x, y) = m_0(r), \quad r = \sqrt{x^2 + y^2}$$



Magnetic nanodots: vortex state

Magnetization **curling** as the lowest energy state for the micro-magnetic particle with size R larger than the single-domain radius.



Disk-shaped particle \implies **vortex state**

[Usov, Peschany, (1993)]

JMMM

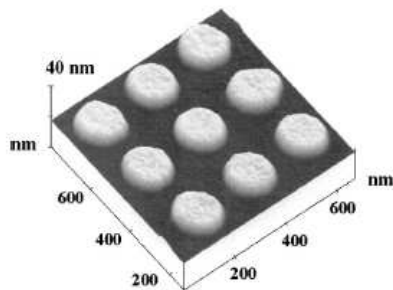


FIG. 1. Atomic force microscopy image of Co dot array with 150 nm diameter and 15 nm thickness, obtained by nanoimprint lithography and lift-off process.

[Lebib et al, J. Appl. Phys. 89 (2001) 3892]

Magnetic nanodots: vortex structure

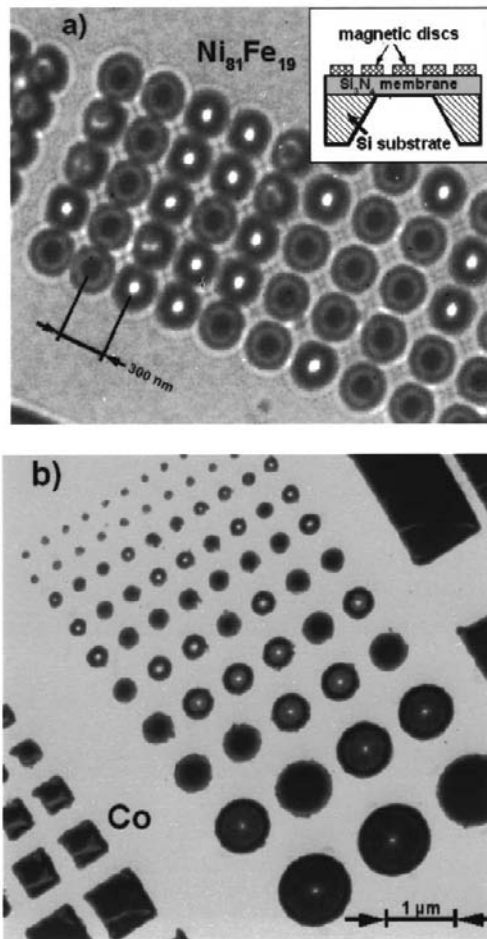


FIG. 1. Fresnel image of (a) permalloy and (b) cobalt disks on a Si_3N_4 membrane. Their respective diameters are (a) 200 nm and (b) between 80 nm and 1 μm . The dark or bright spot in the center of the disks reflects the vortex structure of the magnetization oriented clockwise or counter-clockwise, respectively. The outer ring structures are due to Fresnel fringes (Ref. 6).

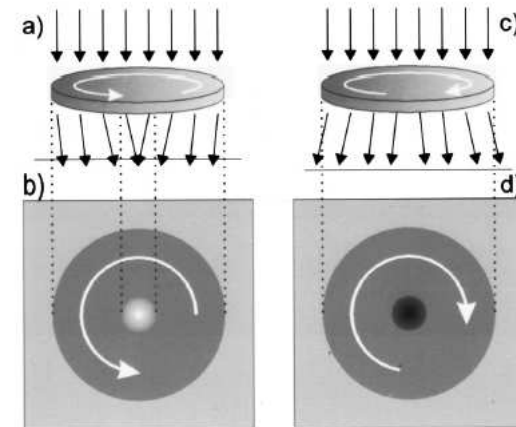
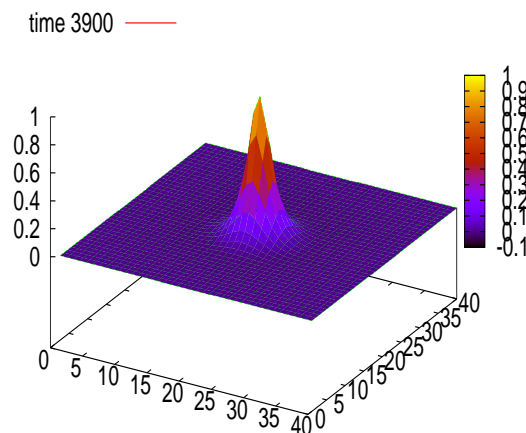
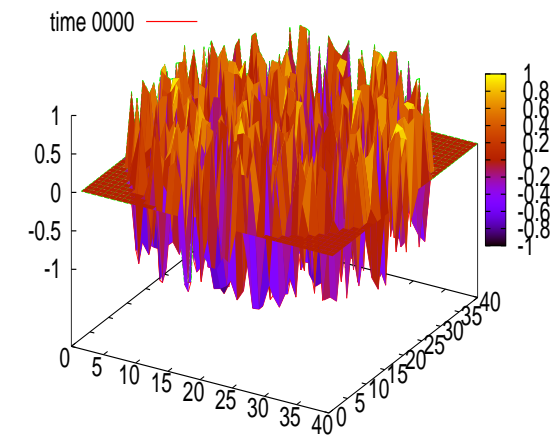


FIG. 2. Origin of the dark and bright spot in the Lorentz micrographs of the disks. While in (a), the counterclockwise orientation of the magnetization vortex focuses the incoming electron beam (black arrows), the electron beam is defocused below the specimen for a clockwise orientation (c). The corresponding intensity distribution in a defocused plane is shown in (b) and (d), respectively. Edge effects due to interference (Fresnel fringes) are not shown.

[Raabe et al, J. Appl. Phys. 88 (2000) 4437]

Magnetic nanodots: ground state is the vortex state

The initial state \implies random magnetization distribution



\Leftarrow vortex state is the finite state

Magnetic nanodots: magnetization reversal and hysteresis loop

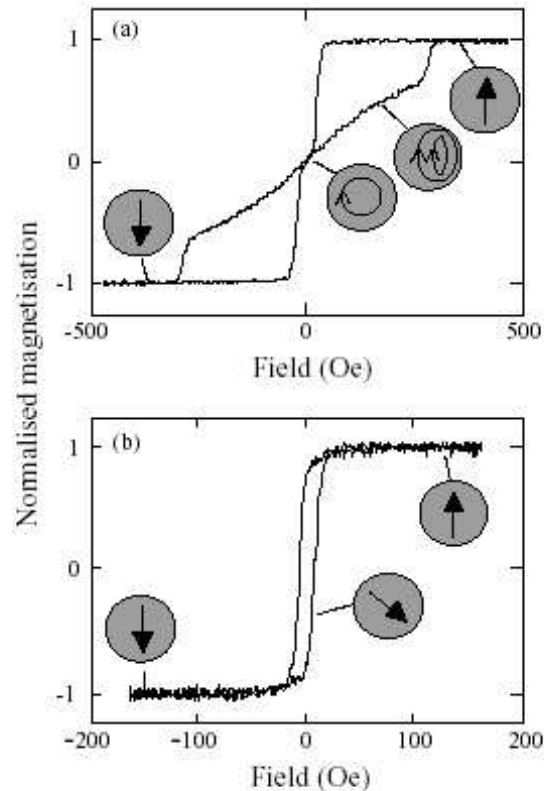


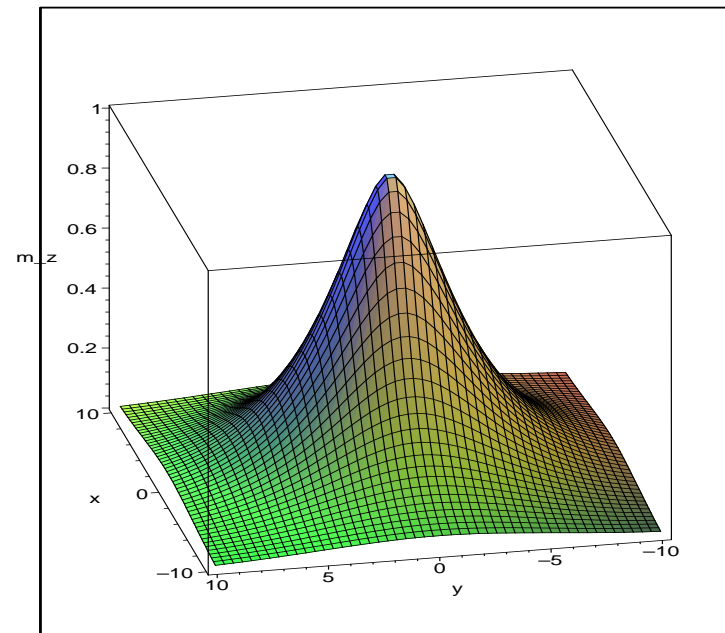
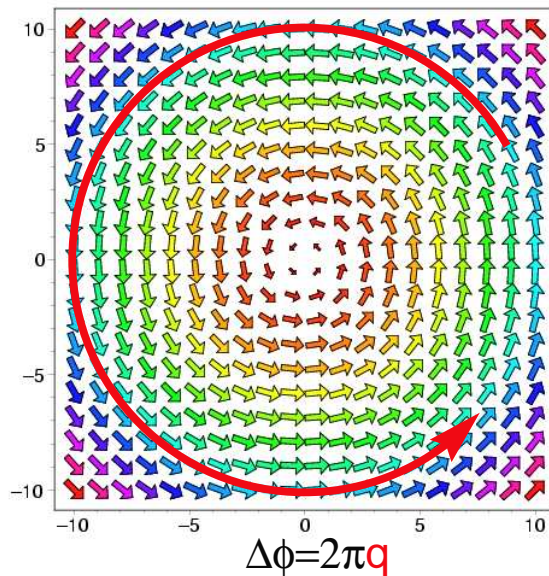
Fig. 1. Hysteresis loops measured from circular nanomagnets of diameter (d) and thickness (t): (a) $d = 300$ nm, $t = 10$ nm; (b) $d = 100$ nm, $t = 10$ nm. The schematic annotation shows the magnetisation within a circular nanomagnet, assuming a field oriented up the page.

[R.P. Cowburn JMMM 242–245
(2002) 505]

Switching of vortex polarization by magnetic fields

$$\mathcal{H} = -\frac{1}{2} \sum_{(\vec{n}, \vec{a})} \left[\vec{S}_{\vec{n}} \cdot \vec{S}_{\vec{n}+\vec{a}} - (1 - \lambda) S_{\vec{n}}^z S_{\vec{n}+\vec{a}}^z \right] - h \sum_{\vec{n}} \left[S_{\vec{n}}^x \cos(\omega t) + S_{\vec{n}}^y \sin(\omega t) \right]$$

$q=1$



Landau-Lifshitz equations

$$\frac{d\vec{S}_{\vec{n}}}{dt} = -\vec{S}_{\vec{n}} \times \frac{\partial \mathcal{H}}{\partial \vec{S}_{\vec{n}}} - \epsilon \frac{\vec{S}_{\vec{n}}}{S} \times \frac{d\vec{S}_{\vec{n}}}{dt}$$

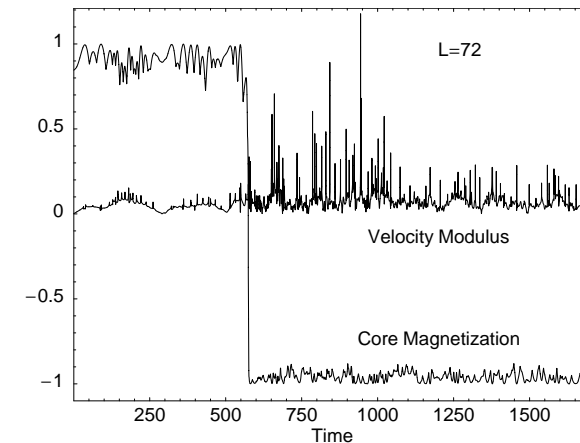
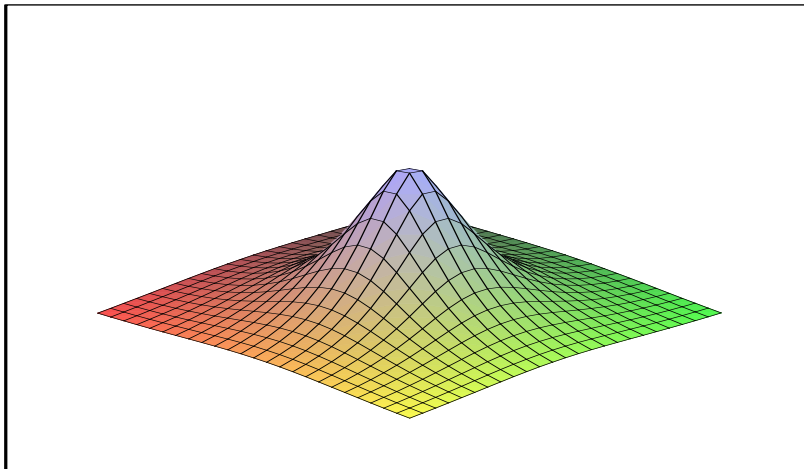
$$n_x, n_y \in (1, N), \quad N = 48, 72, 96, 120$$

$$\lambda = 0.8, 0.9, 0.95, \quad \epsilon = 0.002$$

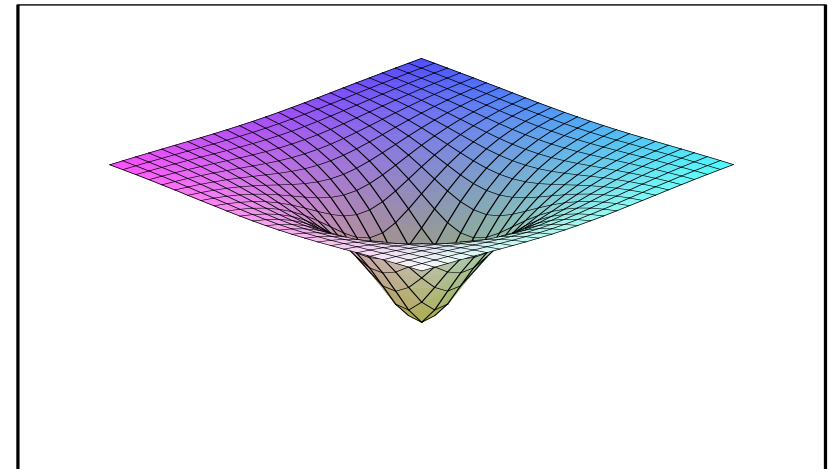
Simulations with rotating magnetic field

Typical time evolution of the core magnetization of a vortex, initially pointing up.

For $\omega = -0.05$, $h = 0.0024$ a flip occurs at $t \sim 574$.
 $p = +1$

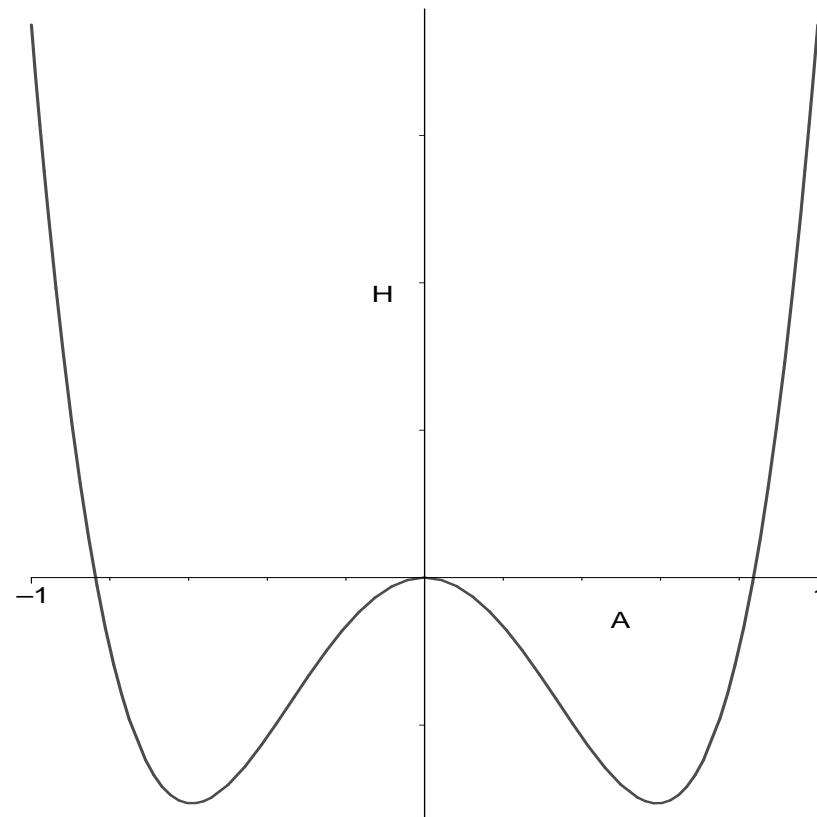
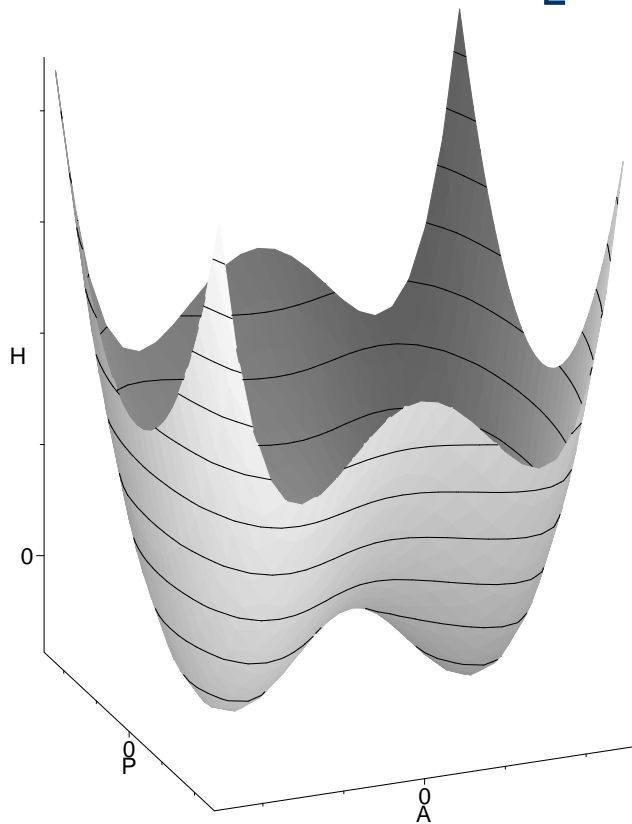


$p = -1$



Discrete core model

$$\mathcal{H}_c = \frac{1}{2} \left[(\lambda_0 - \lambda) A^2 + \frac{1}{4} \lambda_0 A^4 + \lambda_0 P^2 \right]$$

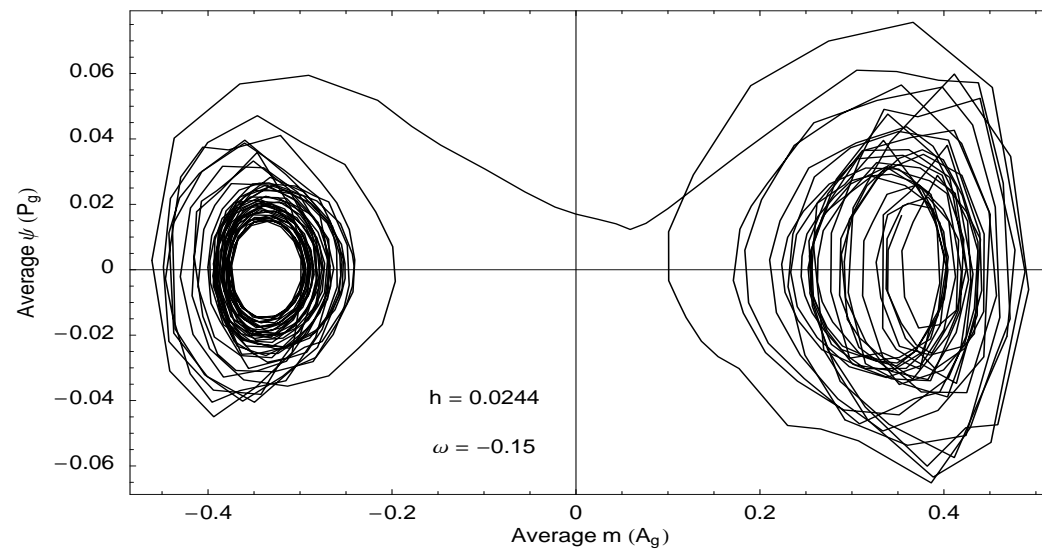
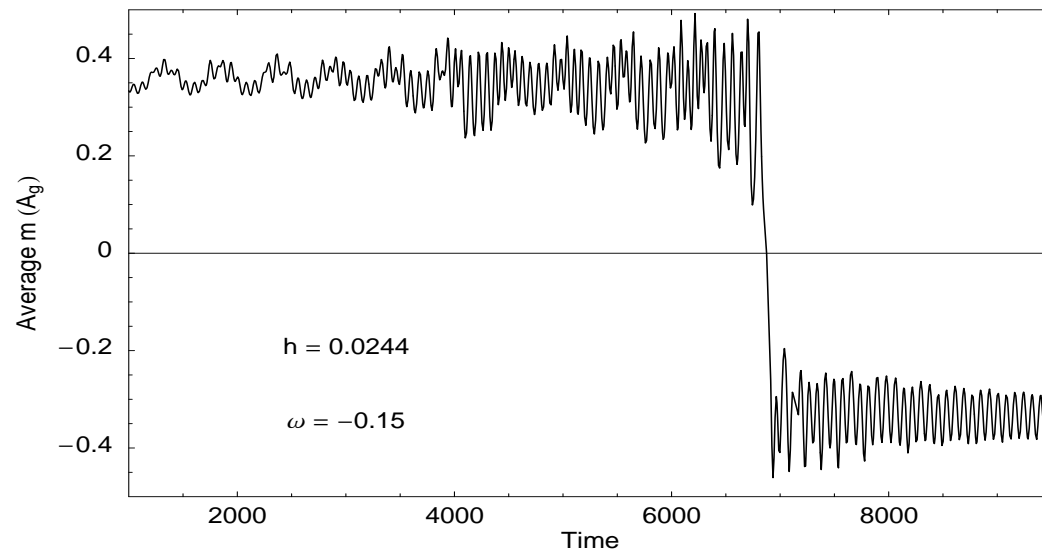


Discrete core model

$$H_u = (\lambda_0 + A)|\chi_+|^2 + (\lambda_0 - A)|\chi_-|^2,$$

$$V(t) = h(i - 1) [(-1 + A - iP)\chi_- + (1 + A + iP)\chi_+^*] e^{i\omega t} + c.c.$$

Discrete core model



Description of the vortex motion

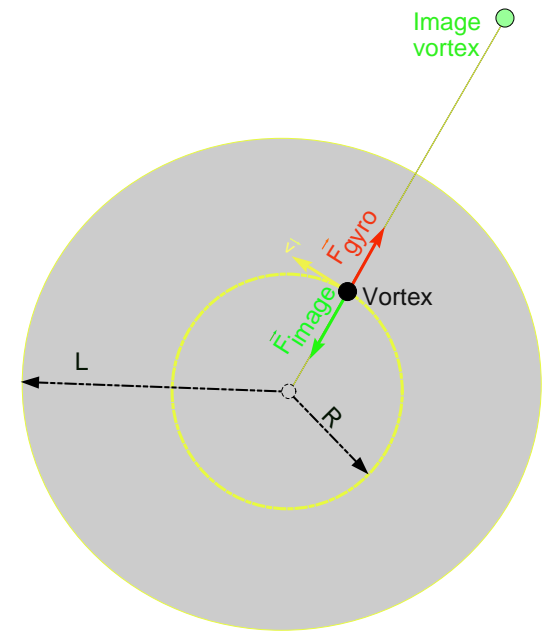
Travelling wave ansatz $\vec{S}(\vec{r}, t) = \vec{S}(\vec{r} - \vec{X}(t))$

Thiele equations: $\vec{F}_{\text{gyro}} + \vec{F}_{\text{image vortex}} = 0$

$$\vec{F}_{\text{gyro}} = G \left[\frac{d\vec{X}}{dt} \times \vec{e}_z \right], \quad G \propto q$$

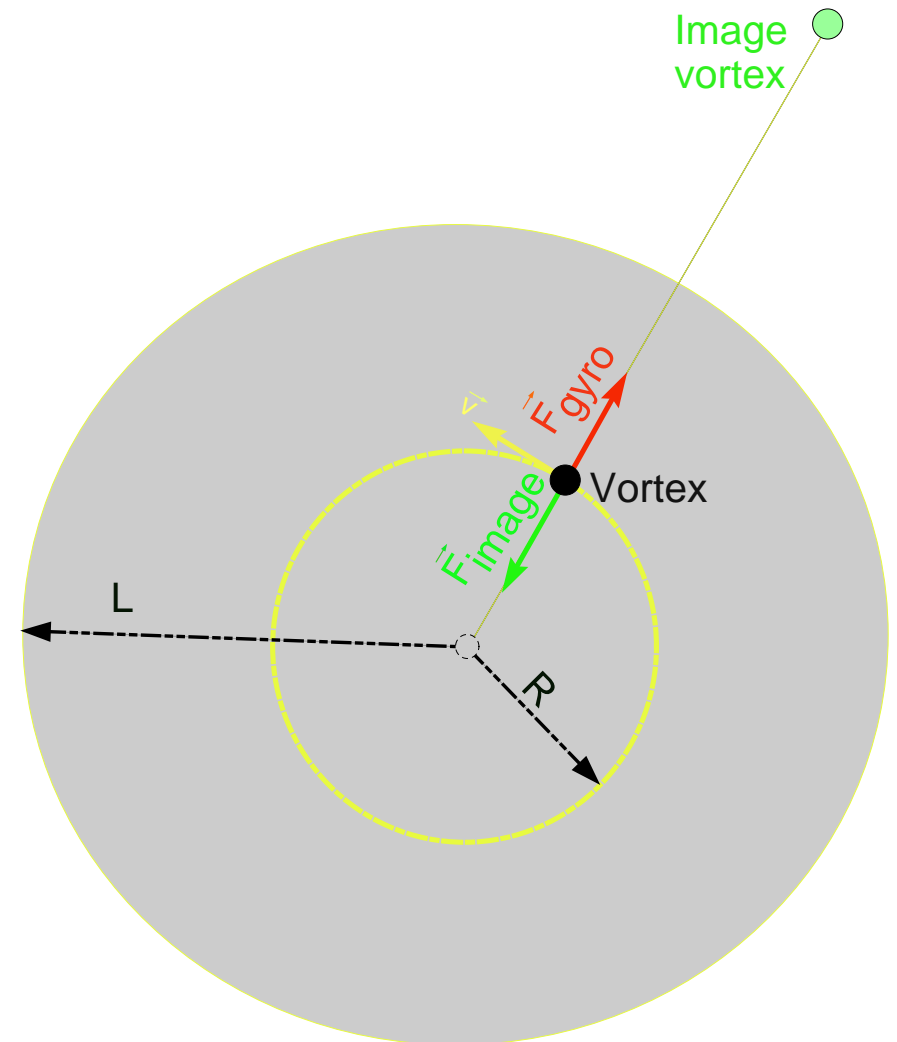
$$\vec{F}_L = \frac{q}{c} \left[\vec{B} \times \frac{d\vec{X}}{dt} \right]$$

$$\vec{F}_{\text{image vortex}} = -\frac{2\pi\vec{X}}{L^2 - R^2}$$



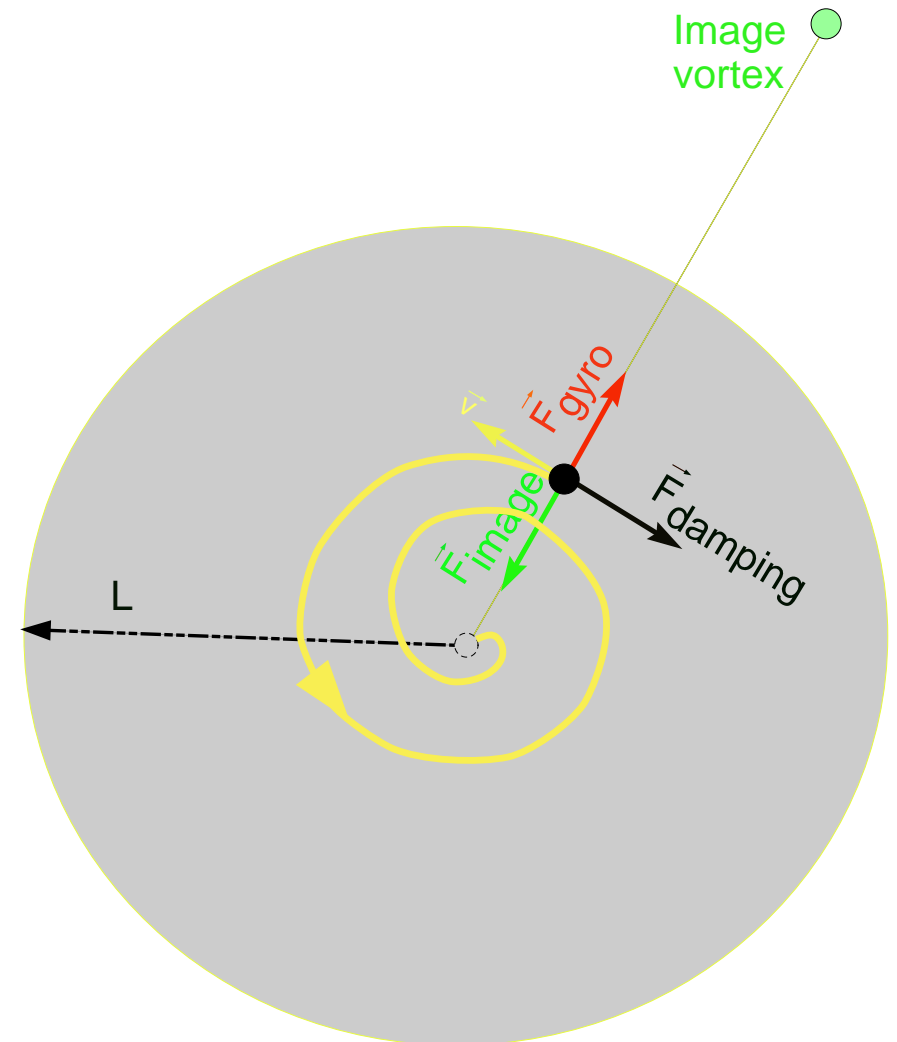
Vortex Motion

Without damping at zero field



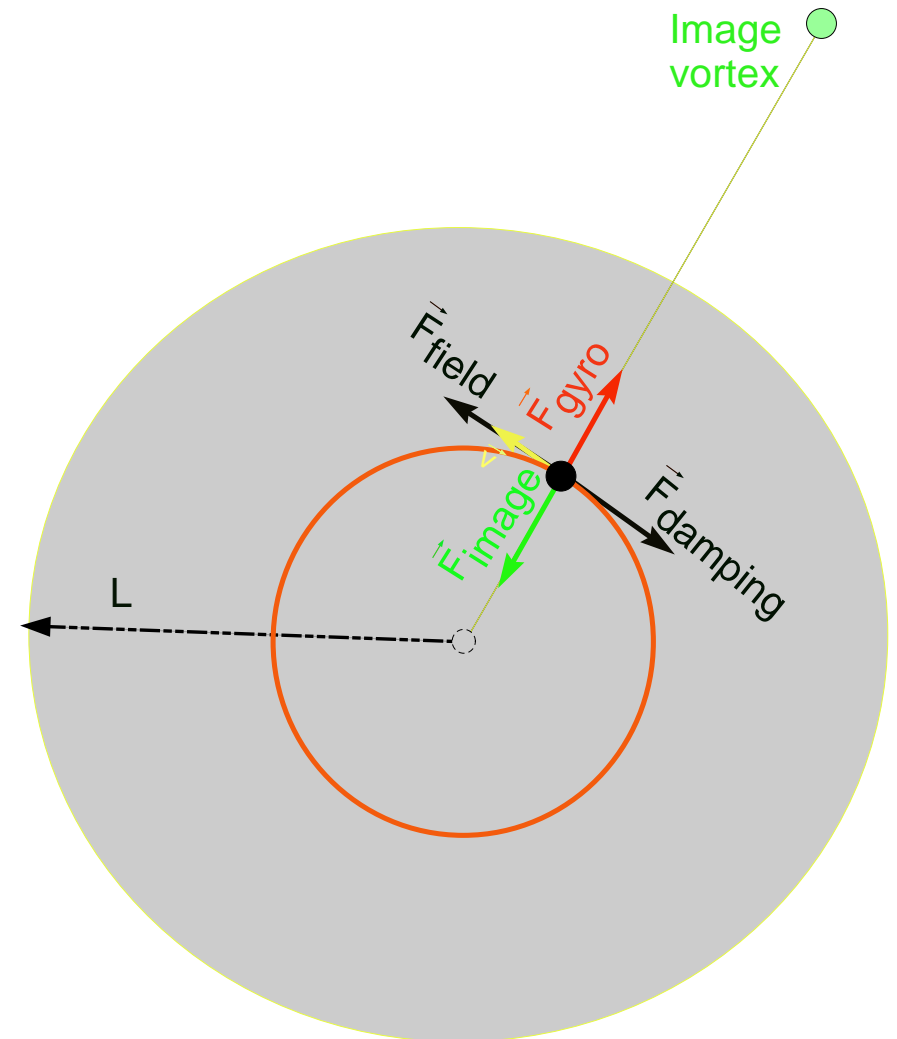
Vortex Motion

With damping at zero field

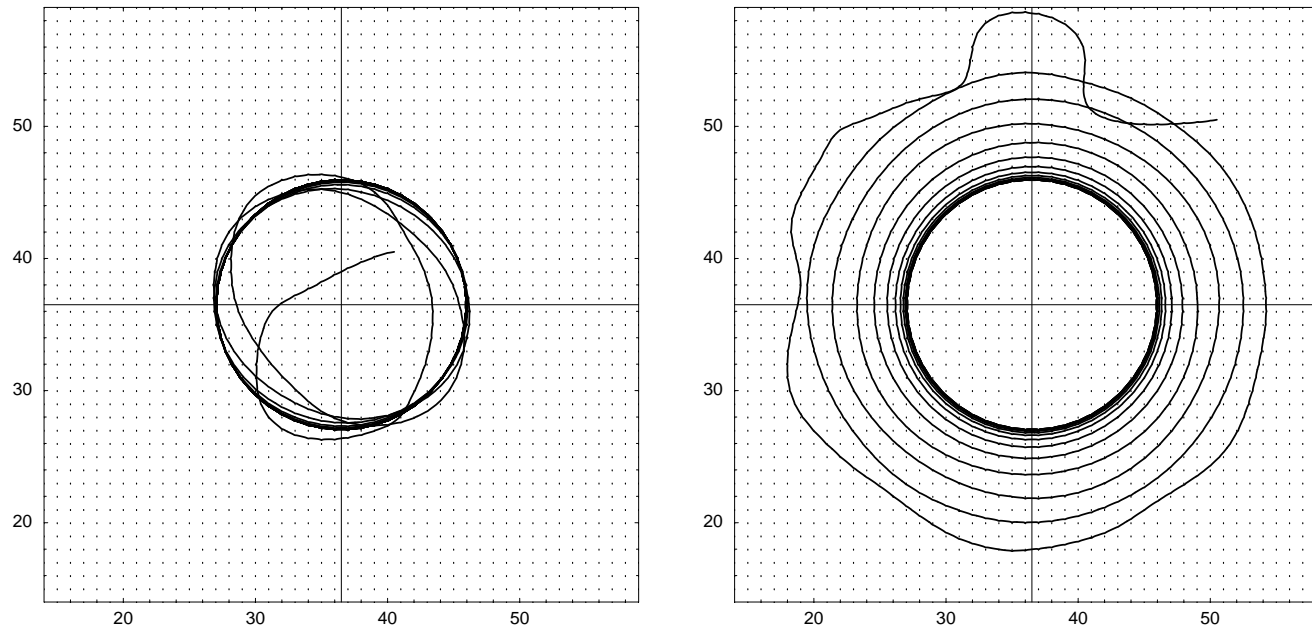


Vortex Motion

With damping and magnetic field



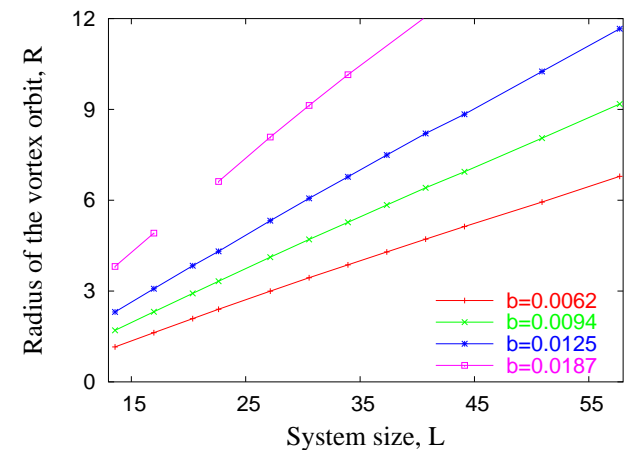
Simulations with rotating magnetic field



Two trajectories of a vortex from simulations of the many-spin model, on a lattice of radius $L = 36$, with a rotating field ($\omega = 0.06$, $h = 0.001$). For this field, all trajectories converge to the same circle independently of the vortex's initial position.

Circular limit cycles

Radius of the vortex orbit R vs. the system radius L , in the circular limit cycle, for a fixed field frequency $\omega = 0.094$ and several amplitudes h .



New collective variable approach

Standard Thiele approach

$$m(z) = m_0 \left[\frac{|z - Z(t)|}{l_0} \right], \quad z = x + iy$$
$$\phi(z) = \varphi_0 + \arg(z - Z(t)) - \arg(z - \bar{Z}(t)) + \arg Z(t).$$

Collective variables: $Z(t) = R(t) \exp i\Phi(t)$ — center of mass

New approach

$$m(z) = m_0 \left[\frac{|z - Z(t)|}{l(t)} \right],$$
$$\phi(z) = \varphi_0 + \arg(z - Z(t)) - \arg(z - \bar{Z}(t)) + \arg Z(t) + \Psi(t).$$

Out-of-plane-component of the total magnetization

$$M(t) = \frac{1}{\pi} \int d^2x m(z, t) = M_0 l^2(t)$$

Effective Lagrangian

Microscopic Lagrangian

$$\mathcal{L} = \sum_{\vec{n}} (1 - m_{\vec{n}}) \frac{d\phi_{\vec{n}}}{dt} - \mathcal{H}$$

Effective Lagrangian:

$$\mathcal{L} \approx M_0 N \dot{\Psi} - R^2 \dot{\Phi} + \frac{R^2}{L^2} - \frac{N^2}{4} - bRL \cos(\Phi + \Psi - vt),$$

$$N = \frac{M - M_0}{M_0}$$

Effective Equations of Motion

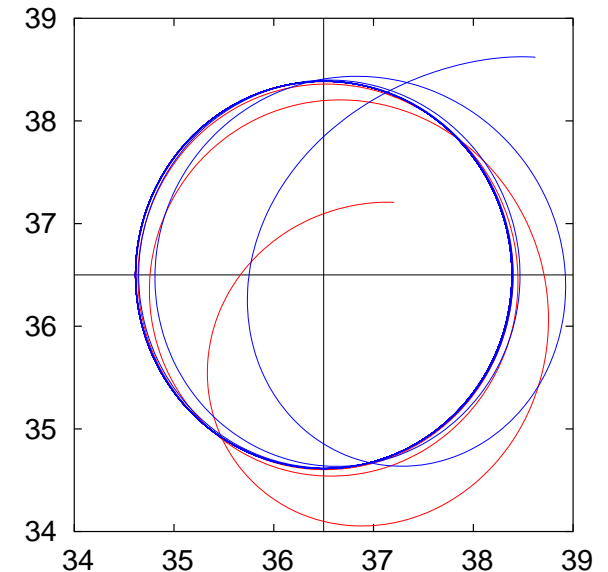
$$\frac{\partial \mathcal{L}}{\partial X_i} - \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{X}_i} \right) = \frac{\partial \mathcal{F}}{\partial \dot{X}_i} .$$

$$X_i = \{R(t), \Phi(t), N(t), \Psi(t)\}$$

$$\begin{cases} \dot{R} = \eta R \dot{\Phi} - \frac{bL}{2} \sin \Delta + \varepsilon \frac{R}{2} \dot{\Psi}, \\ \dot{\Phi} = \frac{1}{L^2} - \eta \frac{\dot{R}}{R} - \frac{bL}{2R} \cos \Delta, \\ M_0 \dot{N} = -\varepsilon L^2 \dot{\Psi} + bRL \sin \Delta - \varepsilon R^2 \dot{\Phi}, \\ 2M_0 \dot{\Psi} = N, \end{cases}$$

$$\Delta = (\Phi + \Psi - vt),$$

$$\eta = \frac{\varepsilon}{2} \ln L$$



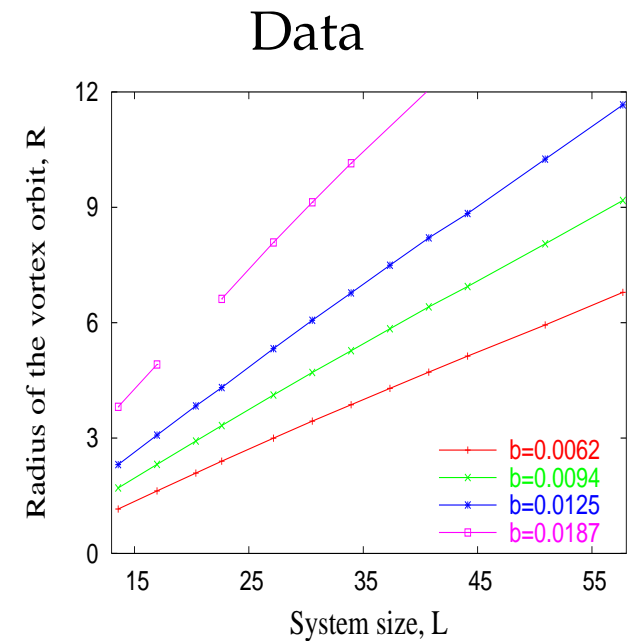
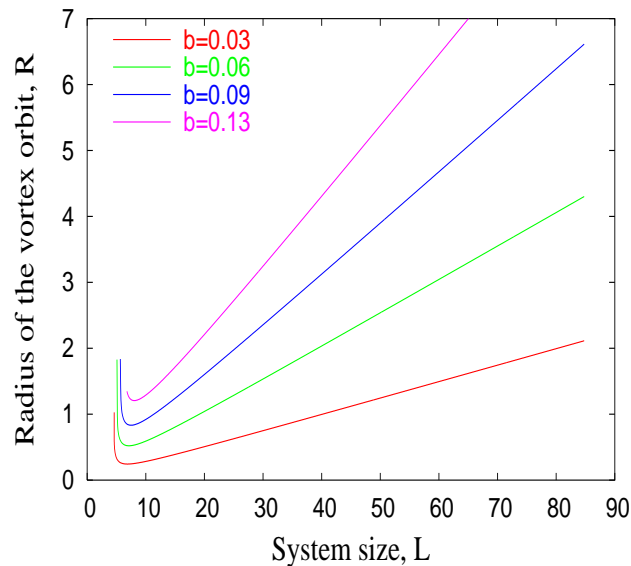
Limit cycle from the model

Limit cycle:

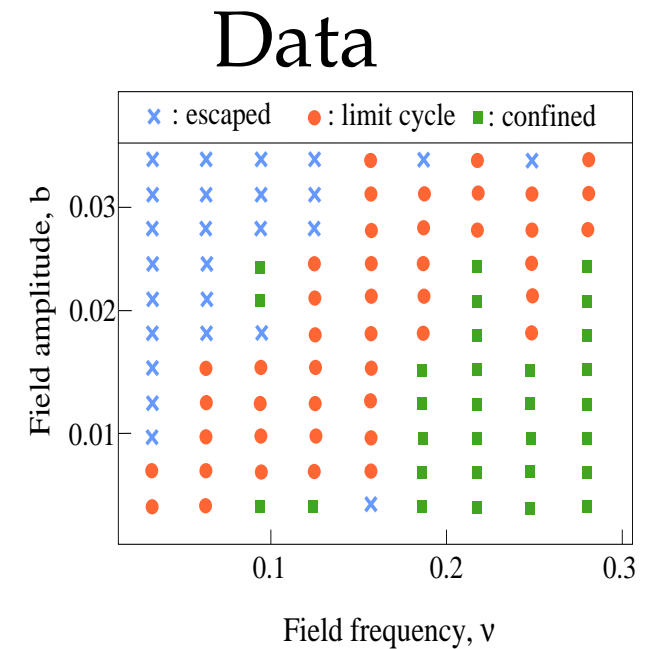
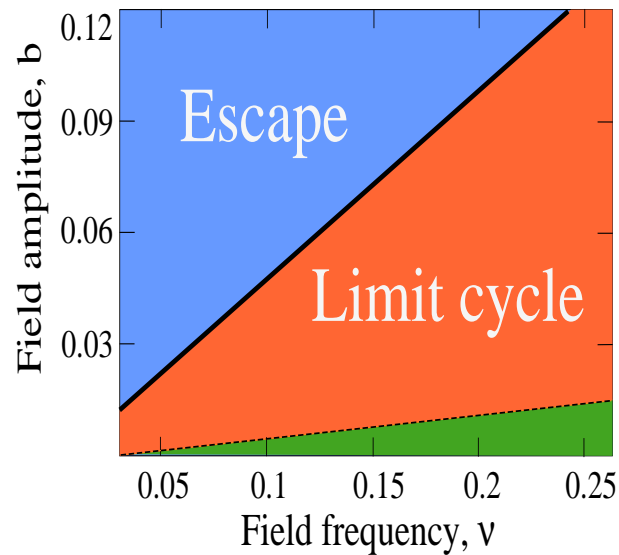
$$\dot{R} = \dot{N} = 0$$

$$\dot{\Phi} + \dot{\Psi} = \nu$$

$$\Rightarrow R \approx \frac{bL}{2\nu}$$



Different types of trajectories



$$\frac{2a}{l_0 L} < \frac{b}{\omega} < \frac{1}{\sqrt{C + \ln L}}$$

Conclusions

- A **new** way of engineering curly magnetized nanostructures is found. It is shown that by applying circularly polarized alternating magnetic fields one can effectively control the magnetization state of magnetic nanodots.

Conclusions

- The vortex magnetization reversal takes place under the action of an ac rotating field. Flips occur when the polarization of the vortex is **anti-parallel** to the angular velocity $\vec{\omega}$. Flips are uni-directional. Flip times depend on the intensity and the frequency of the magnetic field. Flip times weakly depend on the size of the nanodot L .

Conclusions

- When the polarization of the vortex is **parallel** to the angular frequency a non-planar vortex executes a periodic orbital motion: **stable limit cycle**.
- The limit cycle exists in a significant range of field intensity h and frequency ω . The radius of the orbital motion is proportional to a nanodot size L .

Conclusions

- A new analytical approach is developed. It is based on a self-consistent consideration of internal degrees of freedom of vortices. It is very general and can be employed for the description of dynamics of **different 2D solitons** and **vortices** with **complex internal structures**.

Publications

- J.P. Zagorodny, Yu. Gaididei, D.D. Sheka, J.G. Caputo, F.G. Mertens, *Importance of the Internal Shape Mode in Magnetic Vortex Dynamics*, Phys. Rev. Lett. **93**, 167201 (2004)
- H. Büttner, Yu. Gaididei, A. Saxena, T. Lookman and A.R. Bishop, *Domain wall junctions as vortices. Static structure*, J. Phys. A.: Math. Gen. **37**, 8595 (2004).

Publications

- D.D. Sheka, J.G. Caputo, Yu. Gaididei, J.P. Zagorodny, F.G. Mertens, *Vortex motion in a finite-size easy-plane ferromagnet, applied to a nanodot*, Phys. Rev. B **71**, 134420 (2005).
- D.D. Sheka, Yu. Gaididei, J.G. Caputo, J.P. Zagorodny, F.G. Mertens, *Limit cycle for a magnetic vortex dynamics in 2D nanodot*, Ukr. Phys. J. **50**, N 11 (2005).