

# "Breathers" cuánticos

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# Resumen

- Introducción
- "Breathers" cuánticos. Modelos de Hubbard.
- Problemas de cómputo.
  - Programas de manipulación simbólica.
  - Cálculo de espectros.
- Algunos resultados.
  - Sistemas invariantes bajo rotaciones/traslaciones.
  - Sistemas no invariantes bajo rotaciones/traslaciones.
- Ejemplo: Efecto Aharonov-Bohm en un excitón.

# Introducción

## "Breathers" en sistemas "clásicos"

- Sistemas de osciladores acoplados no lineales:

$$H = \sum_{\vec{n}} \left( \frac{1}{2} m_n \dot{\vec{u}}_{\vec{n}}^2 + V_{\vec{n}}(\vec{u}_{\vec{n}}) + C \sum_{\vec{m}} W_{\vec{n},\vec{m}}(\vec{u}_{\vec{n}}, \vec{u}_{\vec{n}+\vec{m}}) \right)$$

- Localización espacial debida a efectos no lineales. Existencia y estabilidad bajo condiciones muy generales.
- Resultados analíticos.
- "Breathers" estáticos y móviles.
- Evidencias experimentales.

# "Breathers" cuánticos

Equivalente cuántico de un "breather" clásico?. Problema sin resolver

- Modelos de Hubbard (fermiones o bosones). Sistemas QDNLS.

$$\hat{H} = - \sum_{j=1}^{f^D} \frac{\gamma_j}{2} b_j^\dagger b_j^\dagger b_j b_j - \sum_{j=1}^{f^D} \sum_p \epsilon_{jp} b_j^\dagger b_{j+p}.$$

Índices  $j$  y  $p$  indican los nodos de una red (D dimensiones).

- $b_j^\dagger$  and  $b_j$  Operadores creación/destrucción (bosones o fermiones),
- $\hat{H}$  conserva el número de "cuantos".  $\hat{N} = \sum_{j=1}^f b_j^\dagger b_j.$

# Método del "Número de estados"

- Problema: Espectro (total o parcial) del operador Hamiltoniano. Diagonalización parcial para un número dado de "cuantos"  $N$ .
- Actuación de los operadores  $b_j$  y  $b_j^\dagger$  en una base  $|\psi_n\rangle = [n_1, n_2, \dots, n_f]$ ,  $N = \sum n_i$ .
- Función de onda:  $|\Psi_n\rangle = \sum_n a_n |\psi_n\rangle$ .  
Ejemplo: problema en una dimensión (cadena): Red de 4 nodos y 7 bosones: [2,0,2,3].



# Problemas de cómputo

- Para un número dado de "cuantos", determinar la representación matricial del operador Hamiltoniano. Problema: Número de vectores de la base crece rápidamente con  $n$  y  $f$ .
- Cálculo de autovectores y autovalores de una matriz (hermítica).
- En sistemas con simetrías adicionales (traslación, rotación,...) el problema se simplifica.

# Algunos resultados

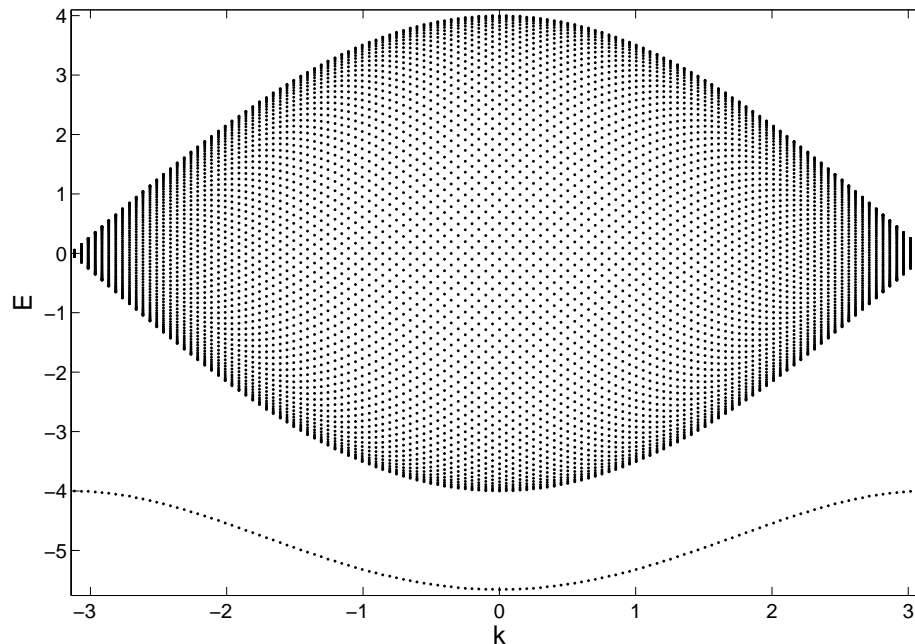
Sistemas invariantes bajo traslaciones (cristales perfectos). Momento definido (número de ondas  $\vec{k}$ ).

- Resultados analíticos
- En general, si el parámetro  $\gamma$  (no linealidad) es suficientemente grande, aparece una estructura de bandas, y una banda aislada correspondiente a estados localizados, en los que existe una alta probabilidad de que los "cuantos" estén juntos en el mismo sitio, pero con igual probabilidad en cualquier punto de la red.

Sistemas no invariantes bajo traslaciones (cristales con impurezas, inhomogeneidades, ...). Localización espacial.

# Espectro

Ejemplo: Autovalores de la energía  $E(k)$  como función del momento  $k$ . QDNLS, red de bosones en una dimensión,  $f = 125$  y  $n = 2$ .



Estado fundamental para  $k = 0$  (no normalizado):

$$|\Psi\rangle = [20 \dots 0] + [020 \dots 0] + \dots + [0 \dots 02] + O(\gamma^{-1})..$$



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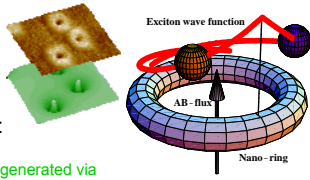


## Abstract

We study the Aharonov-Bohm effect for an exciton on a nanoring using a 2D attractive fermionic Hubbard model. We extend previous results obtained for a 1D ring, in which only azimuthal motion is considered, to a more general case of 2D annular lattices. In general, we show that the existence of the localization effect, increased by the nonlinearity, makes the phenomenon in the 2D system similar to the 1D case. However, the introduction of radial motion introduces extra frequencies, different from the original isolated frequency corresponding to the excitonic Aharonov-Bohm oscillations. If the circumference of the system becomes large enough, the Aharonov-Bohm effect is suppressed.

## The excitonic AB effect for the nano-rings

Nano-sized rings with radius of 30-50nm exist:

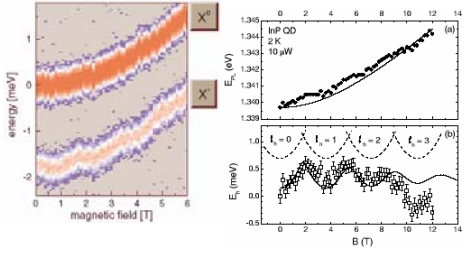


Excitons are being generated via photoluminescence. What about Aharonov-Bohm effect for this nano-geometry and neutral (quasi-)particle?

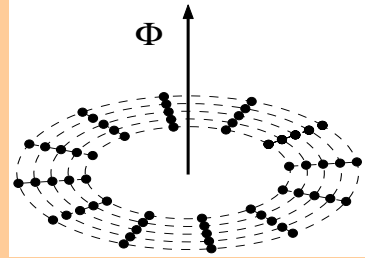
## Experiments

initially for **trions**  
 [Bayer, et al., PRL 90, 186801 (2003)]

recently also for **excitons**:  
 [E. Ribeiro et al., PRL 92, 126401 (2004)]



## The finite-width model: attractive Hubbard on many rings



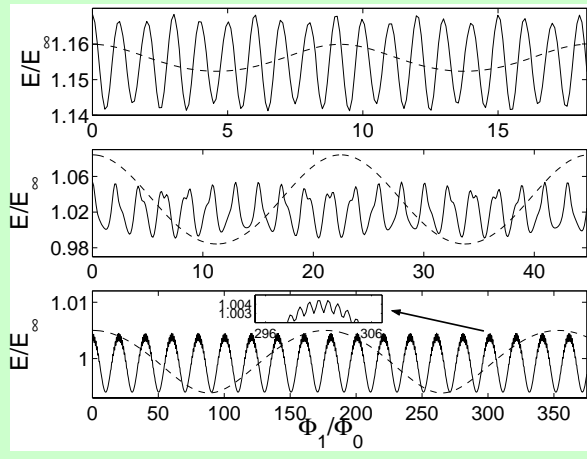
- N rings, M sites on each rings
- Attractive Hubbard interaction for electron/hole pairs

$$H = \sum_{n=1}^N \sum_{m=1}^M t_n^{\parallel} (e^{2\pi i \phi_n / M} \text{electron/hole along ring } n + \text{h.c.}) + \sum_{n=1}^N \sum_{m=1}^M t_n^{\perp} (\text{electron/hole at } m \text{ between rings}) - \gamma \sum_{n=1}^N \sum_{m=1}^M \text{electron-hole onsite interaction at } n, m$$

• Distance dependent hopping:  $t_n^{\perp} = \frac{1}{(r_{n+1} - r_n)^2}$ ,  $t_n^{\parallel} = \frac{1}{4r_n^2 \sin^2(\pi/M)}$

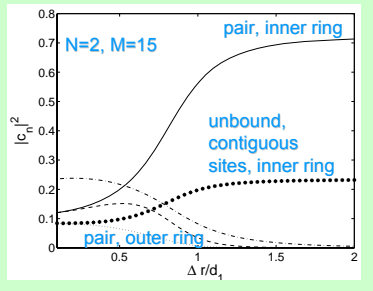
## The two-ring case as example

• AB oscillations

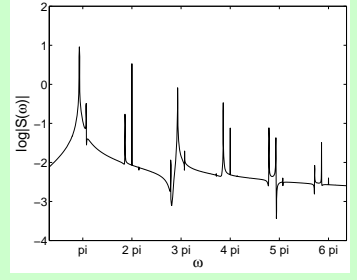


- inter-ring separation is 1.5 times (top), 0.5 (centre) and 0.1 times (bottom) intra-ring spacing
- N=5 rings, continuous line is for M=5 and dashed line for M=15
- inset (bottom) shows excitonic/bound AB oscillations on top of long-ranged unbound oscillations

• where is the ground state e-h pair located?



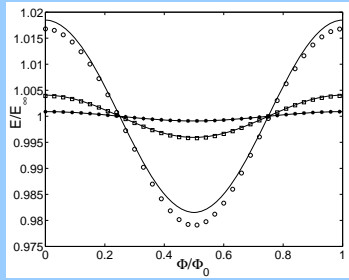
• oscillations for bound and unbound pairs



- Fourier spectrum of ground state oscillations
- N=2 rings, M=5, inter/intra=0.7
- 2π oscillations are excitonic AB osc in inner ring
- other oscs are due to unbound components in other rings

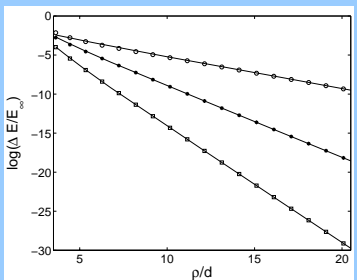
## The single-ring case (again)

• AB oscillations

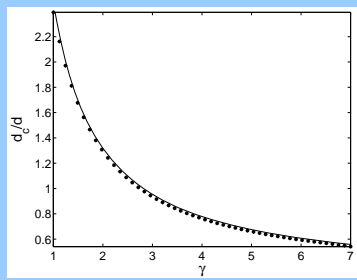


- discrete model reproduces results of continuous solution
- thin lines are perturbative result in γ and ring radius
- oscillations are excitonic AB oscs with period Φ0
- oscillations weaken when either γ or ring radius increase

• radius dependence



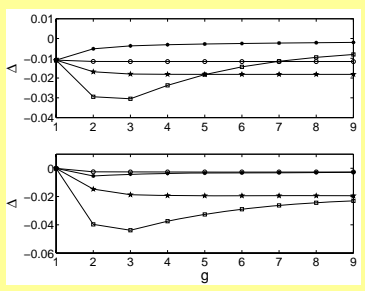
• Interaction strength dependence



## More than two rings

• AB oscillations vs. ring number

- relative amplitude change compared to no magnetic field
- M=5 (top) and 11 (bottom)
- inter/intra=0.1 (asterisks \*), 0.7 (squares), 1.0 (stars) and 1.5 (circles)



- ground state oscillations get weaker as M increases but remain finite!

## Conclusions

AB oscillations survive in a finite ring, but are modified

## References

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